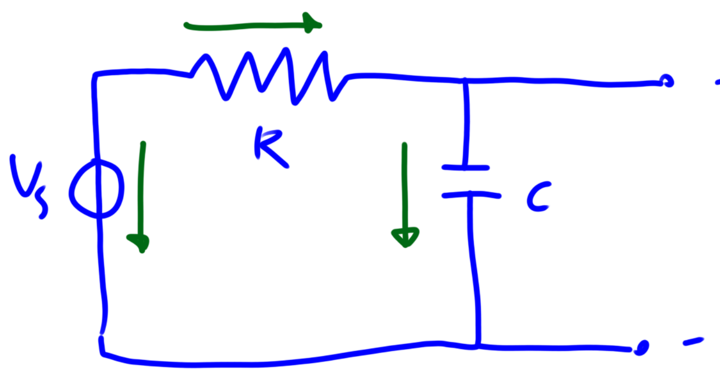
State Space Representation to IO ODE



Solve for V_o

1. Derive ODE directly
2. Linear Graph
   Normal Tree
   SS Model
   Transfer function
   ODE

1. Elemental eqns

$$\frac{dv_c}{dt} = \frac{1}{C} i_c = \frac{1}{C} i_R = \frac{1}{RC} V_R$$

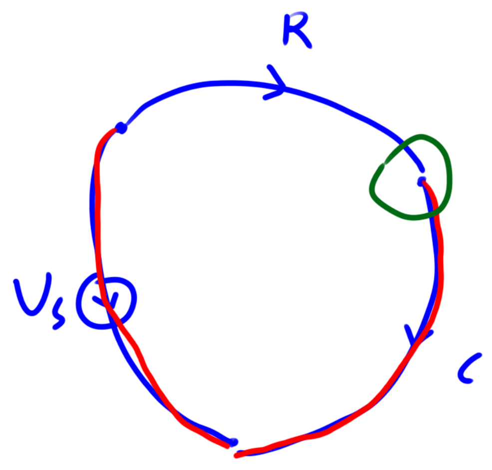
$$V_R = R i_R \qquad = \frac{V_s - V_c}{RC}$$

KCL $i_R = i_c$

KVL $V_s = V_R + V_c \Rightarrow V_R = V_s - V_c$

$$\frac{dv_c}{dt} + \frac{1}{RC} V_c = \frac{1}{RC} V_s$$

2.



Primary: V_s V_c i_R

Secondary: I_s i_c V_R

State: V_c

$x = [V_c]$

$u = [V_s]$

$y = [V_c]$

Elemental

$$\frac{dv_c}{dt} = \frac{1}{C} i_c = \frac{1}{C} i_R = \frac{V_s - V_c}{RC}$$

$$i_R = \frac{1}{R} V_R = \frac{V_s - V_c}{R}$$

Continuity

$i_c = i_R$

compatibility

$V_R = V_s - V_c$

$$\dot{x} = Ax + Bu$$

$$\dot{x} = \left[{}^{-1}/_{RC}\right] x + \left[{}^{1}/_{RC}\right] u$$

$$y = \underbrace{[1]}_{C} x + \underbrace{[0]}_{D} u$$

$$H(s) = C(sI - A)^{-1} B + D$$

$$= 1\left(s(1) - \frac{-1}{RC}\right)^{-1} \frac{1}{RC} + 0$$

$$= \left(s + \frac{1}{RC}\right)^{-1} \frac{1}{RC}$$

$$= \frac{1}{s + \frac{1}{RC}} \frac{1}{RC} = \frac{1}{sRC + 1}$$

$$\frac{RC \frac{dv_c}{dt} + V_c = V_s}{RC} \qquad \overline{RC}$$

$$\frac{dv_c}{dt} + \frac{1}{RC} V_c = \frac{1}{RC} V_s \quad \checkmark$$

$$\frac{1/RC}{s + 1/RC}$$

$$\frac{dv_c}{dt} + \frac{1}{RC} V_c = \frac{1}{RC} V_s$$