6.17. A permanent magnet dc generator is driven by a line around a pulley of radius r as shown in Figure 6.38. One end of the line is connected to a spring of stiffness K, and the other is attached to a source of known force $F(t)$. The line is taut at all times. The generator is connected to a large capacitor C. Derive a set of state equations and an output equation for the voltage across the capacitor. You may ignore the inductance of the generator winding but should include the rotor inertia J and the winding resistance R.

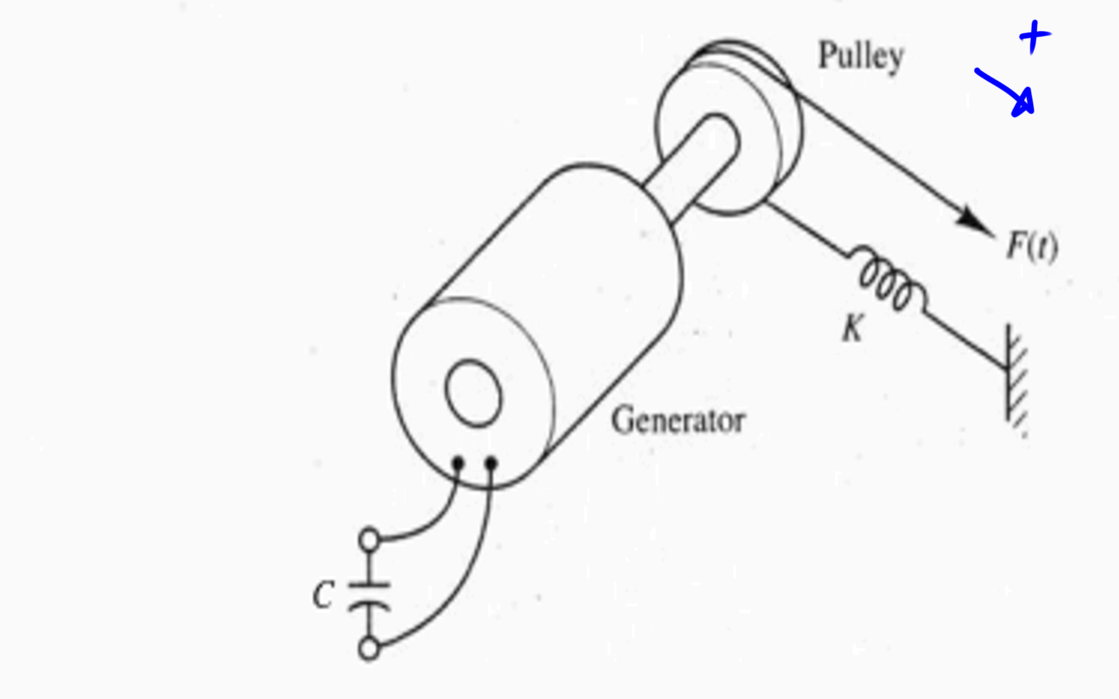
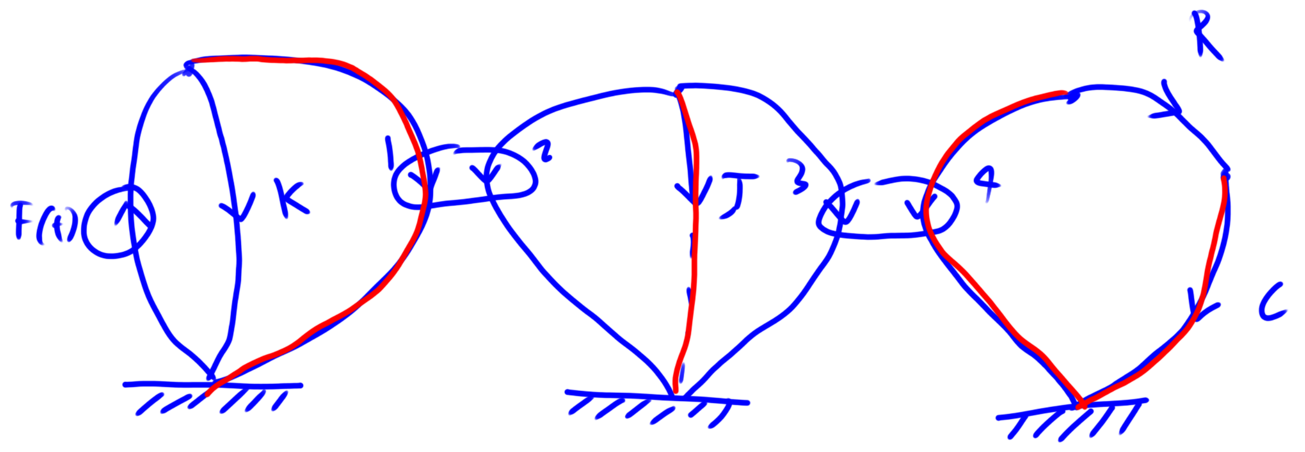


Figure 6.38: A line driven electric generator.

$$\begin{bmatrix} v_4 \\ i_4 \end{bmatrix} = \begin{bmatrix} K_a & 0 \\ 0 & -\frac{1}{K_a} \end{bmatrix} \begin{bmatrix} \Omega_3 \\ \tau_3 \end{bmatrix} \qquad \begin{bmatrix} \Omega_2 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} r & 0 \\ 0 & -\frac{1}{r} \end{bmatrix} \begin{bmatrix} v_1 \\ F_1 \end{bmatrix}$$



Primary: F_s F_K v_1 τ_2 Ω_J τ_3 v_4 i_R v_c

secondary v_s v_K F_1 Ω_2 τ_J Ω_3 i_4 v_R i_c

State: F_K Ω_J v_c $\quad n=3$

Elemental

$$\frac{dF_K}{dt} = K v_K$$

$$v_1 = \frac{1}{r}\Omega_2$$

$$\tau_2 = \frac{-1}{r}F_1$$

$$\frac{d\Omega_J}{dt} = \frac{1}{J}\tau_J$$

$$\tau_3 = -k_a i_4$$

$$v_4 = k_a \Omega_3$$

$$i_R = \frac{1}{R}v_R$$

$$\frac{dv_c}{dt} = \frac{1}{C} i_c$$