6.3. A ball-screw linear actuator is shown in Fig. 6.26. The actuator uses a dc electric motor, with an electromotive coupling constant K_a, armature resistance R, and inductance L, and driven by a voltage source $V_s(t)$. The motor drives a threaded shaft with N threads/cm on which the ball-nut moves linearly. The mechanical load is a mass m which slides on a surface with viscous damping coefficient B.

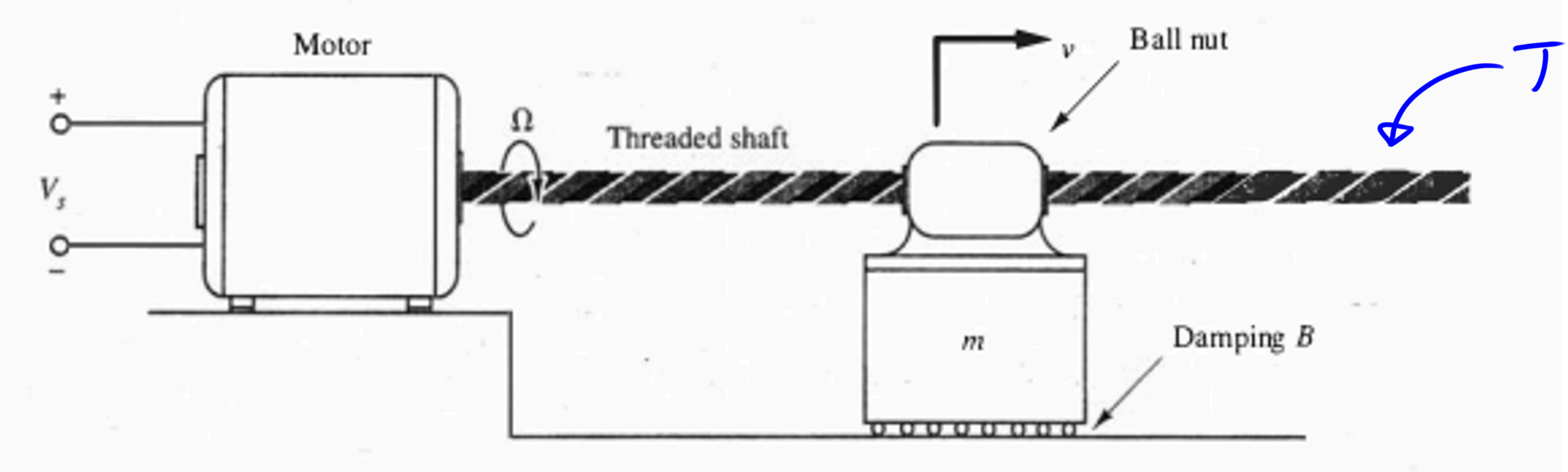
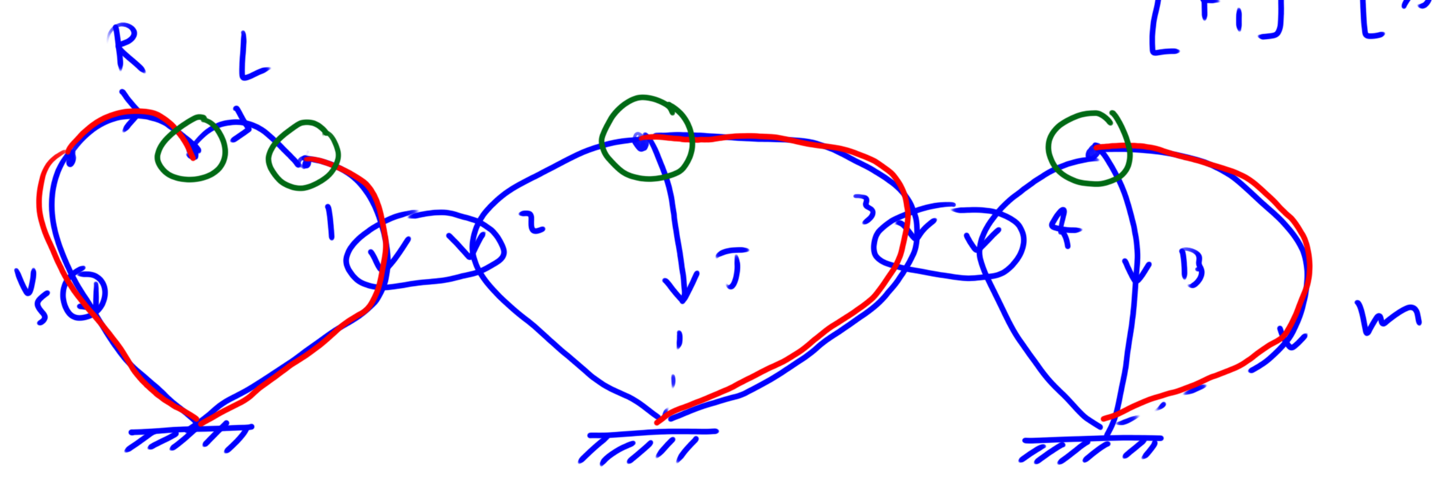


Figure 6.26: A ball-screw drive mechanism.

$$\begin{bmatrix} V_1 \\ F_1 \end{bmatrix} = \begin{bmatrix} TF & 0 \\ 0 & \frac{-1}{TF} \end{bmatrix} \begin{bmatrix} V_2 \\ F_2 \end{bmatrix}$$



$$TF = \frac{\Omega_3}{V_4} \Rightarrow \int \Omega_3 = TF \int V_4$$

$$\theta_3 = TF\, x_4$$

if $\theta_3 = 2\pi$ then

$$x_4 = \frac{1}{N}$$

$$2\pi = TF \frac{1}{N}$$

$$TF = 2\pi N$$

Primary: V_s V_R V_1 i_L Ω_3 τ_2 τ_J V_m F_4 F_B

Secondary: I_s i_R i_1 V_L τ_3 Ω_2 Ω_J F_m V_4 V_B

State: V_m i_L $\qquad n = 2$

Elemental

$$V_R = R i_R = R i_L$$

$$\frac{di_L}{dt} = \frac{1}{L} V_L = \frac{1}{L}(V_s - V_R - V_1) = \frac{1}{L}(V_s - R i_L - K_a \Omega_3)$$

$$= \frac{1}{L}(V_s - R i_L - 2\pi N K_a V_m)$$

$$\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} K_a & 0 \\ 0 & \frac{-1}{K_a} \end{bmatrix} \begin{bmatrix} \Omega_2 \\ \tau_2 \end{bmatrix}$$

$$V_1 = K_a \Omega_2 = K_a \Omega_3$$

$$\tau_2 = -K_a i_1 = -K_a i_L$$

$$\tau_J = J \frac{d\Omega_J}{dt} = J \frac{d\Omega_3}{dt}$$

$$\begin{bmatrix} \Omega_3 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} 2\pi N & 0 \\ 0 & -1/2\pi N \end{bmatrix} \begin{bmatrix} V_4 \\ F_4 \end{bmatrix}$$

$$\Omega_3 = 2\pi N V_4 = 2\pi N V_m$$

$$F_4 = -2\pi N \tau_3 = 2\pi N(\tau_2 + \tau_J)$$

$$F_B = B V_B = B V_m$$

$$\frac{dV_m}{dt} = \frac{1}{m} F_m = \frac{-1}{m}(F_4 + F_B) = \frac{-1}{m}(2\pi N(\tau_2 + \tau_J) + B V_m)$$

$$= \frac{-1}{m}\left(2\pi N\left(-K_a i_L + J\frac{d\Omega_3}{dt}\right) + B V_m\right)$$

$$= \frac{-1}{m}\left(2\pi N\left(-K_a i_L + 2\pi N J\frac{dV_m}{dt}\right) + B V_m\right)$$

$$= \frac{2\pi N K_a}{m} i_L - \frac{4\pi^2 N^2 J}{m}\frac{dV_m}{dt} - \frac{B}{m} V_m$$

$$\frac{dV_m}{dt} + \frac{4\pi^2 N^2 J}{m}\frac{dV_m}{dt} = \frac{2\pi N K_a}{m} i_L - \frac{B}{m} V_m$$

$$\frac{dV_m}{dt}\left(1 + \frac{4\pi^2 N^2 J}{m}\right) = \frac{2\pi N K_a}{m} i_L - \frac{B}{m} V_m$$

$$\frac{dV_m}{dt}(m + 4\pi^2 N^2 J) = 2\pi N K_a i_L - B V_m$$

$$\frac{dV_m}{dt} = \frac{2\pi N K_a}{m + 4\pi^2 N^2 J} i_L - \frac{B}{m + 4\pi^2 N^2 J} V_m$$

Continuity

$$i_R = i_L$$

$$i_1 = i_L$$

$$\tau_3 = -\tau_2 - \tau_J$$

$$F_m = -F_4 - F_B$$

Compatibity

$$V_L = V_s - V_R - V_1$$

$$\Omega_2 = \Omega_3$$

$$\Omega_J = \Omega_3$$

$$V_4 = V_m$$

$$V_B = V_m$$

$$x = \begin{bmatrix} i_L \\ V_m \end{bmatrix} \qquad u = [V_s] \qquad y = [V_m]$$

$$\dot{x} = \begin{bmatrix} -R/L & \frac{-2\pi N K_a}{L} \\ \frac{2\pi N K_a}{m + 4\pi^2 N^2 J} & \frac{-B}{m + 4\pi^2 N^2 J} \end{bmatrix} x + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} u$$

$$y = [0 \quad 1]\, x + [0]\, u$$