emech.exe Exercises for Chapter emech

Exercise emech.triangle

Respond to the following questions and imperatives with one or two sentences and, if needed, equations and/or sketch.

a. Why do we include a resistor in lumped-parameter motor models?
b. How are brushes used in brushed DC motors?
c. With regard to standard motor curves, why do we say the "braking power" is equivalent to the power that could be successfully transferred by the motor to the mechanical system?
d. In terms of electrical and mechanical processes, why does an efficiency versus torque motor curve have a peak?
e. As a DC motor's bearings wear down, how will its efficiency curve be affected?

Exercise emech.square

Consider the system presented in the schematic of Fig. exe.1. Let the DC motor have motor constant K_a (units N-m/A) and let the motor be driven by an ideal current source I_S. Assume the motor inertia has been lumped into J_1 and motor damping lumped into B_1.

a. Draw a linear graph model.
b. Draw a normal tree.
c. Identify any dependent energy storage elements. If the motor was driven by an ideal voltage source instead, how would this change?

Draw a linear graph model and normal tree.

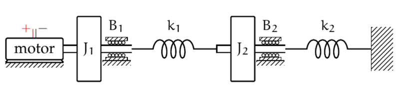
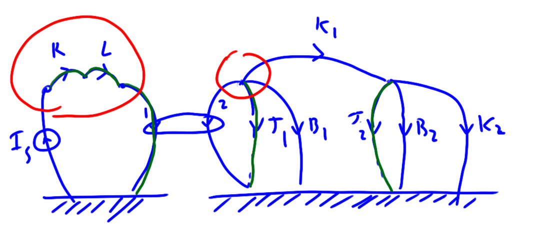


Figure exe.1: schematic of an electromechanical system for Exercise emech..



Primary: I_s v_R v_L v_1 τ_2 Ω_{J_1} τ_{B_1} τ_{k_1} Ω_{J_2} τ_{B_2} τ_{k_2}

Secondary: V_s i_R i_L i_1 Ω_2 τ_{J_1} Ω_{B_1} Ω_{k_1} τ_{J_2} Ω_{B_2} Ω_{k_2}

State: τ_{k_1} τ_{k_2} Ω_{J_1} Ω_{J_2} $n=4$

$$\frac{d\Omega_{J_1}}{dt} = \frac{1}{J_1}\tau_{J_1} = \frac{1}{J_1}(-\tau_2 - \tau_{B_1} - \tau_{k_1}) = \frac{1}{J_1}(K_a I_s - B_1 \Omega_{J_1} - \tau_{k_1})$$

$$\tau_{J_1} = -\tau_2 - \tau_{B_1} - \tau_{k_1}$$

$$\tau_2 = -K_a i_1$$

$$i_1 = I_s$$

$$V_s = V_R + V_L + V_1 = I_s R + \frac{dI_s}{dt} L + K_a \Omega_{J_1}$$

Exercise emech.rectangle

Consider the system presented in the schematic of Fig. exe.1. From the linear graph model and normal tree derived in Exercise emech., derive a state-space model in standard form. Let the outputs be θ_{J_1} and θ_{J_2}, the angular positions of the flywheels.

Exercise emech.quadrilateral

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Consider the linear graph model of a motor coupled to a rotational mechanical system shown in Fig. exe.2. This is similar to the model from the Lec. emech.real, but includes the flexibility of the shaft coupler. An ideal voltage source drives the motor—modeled as an ideal transducer with armature resistance R and inductance L, given by the manufacturer in Table real.1. The ideal transducer's rotational mechanical side (2) is connected to a moment of inertia J_m modeling the rotor inertia and damping B_b modeling the internal motor damping, both values given in the motor specifications. Take $B_b = B_m$ and $J_f = 0.324 \cdot 10^{-3}$ kg-m^2. Assume the shaft coupling has a torsional stiffness of $k = 100$ N-m/rad.

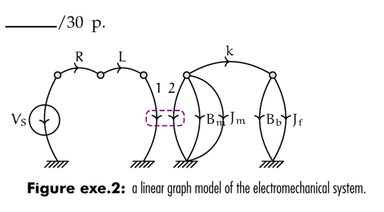

Figure exe.2: a linear graph model of the electromechanical system.

a. Derive a state-space model for the system with outputs i_1 and Ω_{J_f}.
b. Create a Matlab ss model for the system and simulate its response from rest to an input voltage $V_S = 10$ V.
c. Plot the outputs through time until they reach steady state.

Exercise emech.mrpotatohead

_____/30 p.

Consider the linear graph model (with normal tree) of Fig. exe.3. This is a model of a motor with constant K_a connected to a pair of meshing gears with tranformer ratio N, the output over input gear ratio. An ideal voltage source drives the motor—modeled as an ideal transducer with armature resistance R and inductance L. The motor's rotational mechanical side (2) is connected to a moment of inertia J_2 modeling the rotor and drive gear combined inertia. The damping element B_2 models the internal motor damping and the drive gear bearing damping. The output side of the gear transducer (4) is connected to a moment of inertia J_4 modeling the output gear and load combined inertia. The damping element B_4 models the internal motor damping and the drive gear bearing damping. Use the parameter values given in Table exe.1.

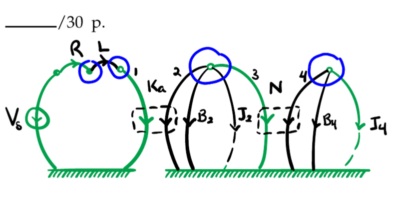

Figure exe.3: a linear graph model with normal tree in green of the electromechanical system of Exercise emech. with a gear reduction.

Primary: V_s v_R i_L v_1 τ_2 τ_{B_2} τ_{J_2} Ω_3 τ_4 τ_{B_4} Ω_{J_4}

Secondary: I_s i_R v_L i_1 Ω_2 Ω_{B_2} Ω_{J_2} τ_3 Ω_4 Ω_{B_4} τ_{J_4}

State: Ω_{J_4} i_L $n=2$

Elemental:

$$V_R = R i_R$$

$$\frac{di_L}{dt} = \frac{1}{L} V_L$$

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} K_a & 0 \\ 0 & -1/K_a \end{bmatrix} \begin{bmatrix} \Omega_2 \\ \tau_2 \end{bmatrix}$$

$$V_1 = K_a \Omega_2$$

$$\tau_2 = -K_a i_1$$

$$\tau_{B_2} = B_2 \Omega_{B_2}$$

$$\tau_{J_2} = J_2 \frac{d\Omega_{J_2}}{dt}$$

$$\begin{bmatrix} \Omega_3 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} N & 0 \\ 0 & -1/N \end{bmatrix} \begin{bmatrix} \Omega_4 \\ \tau_4 \end{bmatrix}$$

$$\Omega_3 = N\Omega_4$$

$$\tau_4 = -N\tau_3$$

$$\tau_{B_4} = B_4 \Omega_{B_4}$$

$$\frac{d\Omega_{J_4}}{dt} = \frac{1}{J_4}\tau_{J_4}$$

Continuity:

$$i_R = i_L$$

$$i_1 = i_L$$

$$\tau_3 = -\tau_2 - \tau_{B_2} - \tau_{J_2}$$

$$\tau_{J_4} = -\tau_4 - \tau_{B_4}$$

Compatability:

$$V_L = V_s - V_R - V_1$$

$$\Omega_2 = \Omega_3$$

$$\Omega_{J_2} = \Omega_3$$

$$\Omega_{B_2} = \Omega_3$$

$$\Omega_4 = \Omega_{J_4}$$

$$\Omega_{B_4} = \Omega_{J_4}$$

Table exe.1: parameter values for Exercise emech..

R	2 Ω
L	8 mH
K_a	0.2 N-m/A
J_2	$0.1 \cdot 10^{-3}$ kg-m^2
B_2	50 μN-m/(rad/s)
N	5
J_4	$1 \cdot 10^{-3}$ kg-m^2
B_4	70 μN-m/(rad/s)

a. Derive a state-space model for the system with outputs Ω_{J_2} and Ω_{J_4}.
b. Create a Matlab ss model for the system and simulate its response from rest to an input voltage $V_S = 20$ V.
c. Plot the outputs through time until they reach steady state.

$$\frac{di_L}{dt} = \frac{1}{L}V_L = \frac{1}{L}(V_s - V_R - V_1) = \frac{1}{L}(V_s - R i_R - K_a \Omega_2) = \frac{1}{L}(V_s - R i_L - K_a \Omega_3) = \frac{1}{L}(V_s - R i_L - K_a N \Omega_4) = \frac{1}{L}(V_s - R i_L - K_a N \Omega_{J_4})$$

$$\frac{d\Omega_{J_4}}{dt} = \frac{1}{J_4}(\tau_{J_4}) = \frac{1}{J_4}(-\tau_4 - \tau_{B_4}) = \frac{1}{J_4}(N\tau_3 - B_4 \Omega_{B_4}) = \frac{1}{J_4}(N(-\tau_2 - \tau_{J_2} - \tau_{B_2}) - B_4 \Omega_{J_4})$$

$$= \frac{1}{J_4}\left(N\left(K_a i_1 - J_2 \frac{d\Omega_{J_2}}{dt} - B_2 \Omega_{B_2}\right) - B_4 \Omega_{J_4}\right)$$

$$= \frac{1}{J_4}\left(N\left(K_a i_L - J_2 \frac{d\Omega_3}{dt} - B_2 \Omega_3\right) - B_4 \Omega_{J_4}\right)$$

$$= \frac{1}{J_4}\left(N\left(K_a i_L - J_2 N \frac{d\Omega_4}{dt} - B_2 \Omega_4\right) - B_4 \Omega_{J_4}\right)$$

$$\frac{d\Omega_{J_4}}{dt} = \frac{1}{J_4}\left(N\left(K_a i_L - J_2 N \frac{d\Omega_{J_4}}{dt} - B_2 \Omega_{J_4}\right) - B_4 \Omega_{J_4}\right)$$

$$= \frac{1}{J_4}\left(N K_a i_L - J_2 N^2 \frac{d\Omega_{J_4}}{dt} - N B_2 \Omega_{J_4} - B_4 \Omega_{J_4}\right)$$

$$\frac{d\Omega_{J_4}}{dt} + \frac{J_2 N^2}{J_4}\frac{d\Omega_{J_4}}{dt} = \frac{1}{J_4}(N K_a i_L - N B_2 \Omega_{J_4} - B_4 \Omega_{J_4})$$

$$\frac{\frac{d\Omega_{J_4}}{dt}\left(1 + \frac{J_2 N^2}{J_4}\right)}{\left(1 + \frac{J_2 N^2}{J_4}\right)} = \frac{\frac{1}{J_4}(N K_a i_L - N B_2 \Omega_{J_4} - B_4 \Omega_{J_4})}{\left(1 + \frac{J_2 N^2}{J_4}\right)}$$

Exercise emech.clunker

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Draw a linear graph, a normal tree, identify state variables, identify system order, and denote any dependent energy storage elements for each of the following schematics.

a. The electronic system of Fig. exe.4, voltage and current sources, and transformer with transformer ratio N.
b. The electromechanical system of Fig. exe.5 with motor model parameters shown, coordinate arrow in green. Model the propeller as a moment of inertia J_2 and damping B_2.
c. The translational mechanical system of Fig. exe.6, force source, coordinate arrow in green.

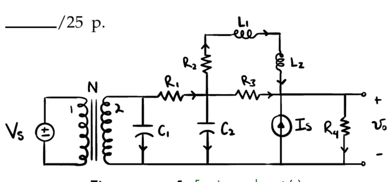


Figure exe.4: Exercise emech. part (a)

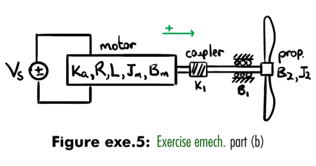


Figure exe.5: Exercise emech. part (b)

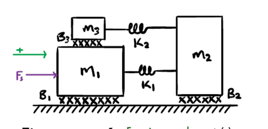
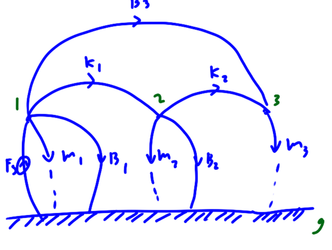


Figure exe.6: Exercise emech. part (c)



Exercise emech.curvy

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Consider the DC motor curves of Fig. curves.2, reproduced in Fig. exe.7.

a. At peak efficiency, what is the steady-state motor speed?
b. At peak efficiency, what is the steady-state motor torque?
c. You are to use this motor to drive a load at a constant angular speed of 100 rad/s with at least 1 N-m of torque. You wisely choose to use a gear reduction between the motor and load. What should the gear ratio be to meet the above requirements and optimize efficiency? Justify your answer in terms of the motor curves of Fig. exe.7.

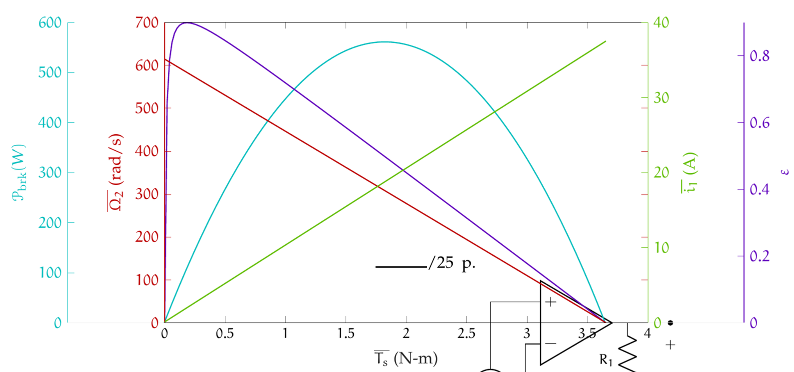

Figure exe.7: the motor curve Fig. curves.2 for Exercise emech..

Exercise emech.chair

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Consider the opamp circuit of Fig. exe.8, which will be used to drive a motor. The input can supply a variable $V_S \in [0, 10]$ V, the motor has constant $K_a = 0.05$ V/(rad/s) and coil resistance $R_m = 1$ Ω, and the opamp has differential supplies ±24 V. Assume the maximum torque magnitude required from the motor at top speed is $|T_2| = 0.1$ N-m and ignore any voltage drop in the motor due to the coil inductance.[14] Select R_1 and R_2 to demonstrably meet the following requirements:

a. drivable motor speeds of at least [0, 400] rad/s,
b. no saturation of the opamp, and
c. a maximum power dissipation by R_1 and R_2 less than 300 mW.

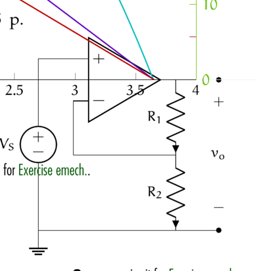

Figure exe.8: opamp circuit for Exercise emech..

14. Do not ignore the voltage drop across R_m, though. Note that this amounts to an assumption of steady-state operation at top speed. By requiring a specific T_2, we are also implicitly ignoring torque losses due to motor bearing damping.

Part II

Time response

lti

Linear time–invariant system properties

1 In this chapter, we will extend our understanding of linear, time-invariant (LTI) system properties. We must keep in mind a few important definitions.

2 The transient response of a system is its response during the initial time-interval during which the initial conditions dominate. The steady-state response of a system is its remaining response, which occurs after the transient response. Fig. lti.1 illustrates these definitions.

transient response

steady–state response

3 The free response of a system is the response of the system to initial conditions—without forcing (i.e. the specific solution of the io ODE with the forcing function identically zero). This is closely related to, but distinct from, the transient response, which is the free response plus an additional term. This additional term is the forced response: the response of the system to a forcing function—without initial conditions (i.e. the specific solution of the io ODE with the initial conditions identically zero).

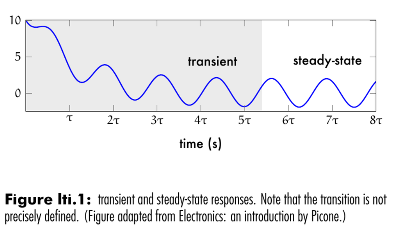

Figure lti.1: transient and steady-state responses. Note that the transition is not precisely defined. (Figure adapted from Electronics: an introduction by Picone.)

free response

forced response