

## com.euler Euler's formulas

Euler's formula is our bridge back-and forth between trigonometric forms ( $\cos \theta$  and  $\sin \theta$ ) and complex exponential form ( $e^{j\theta}$ ):

$$e^{j\theta} = \cos \theta + j \sin \theta. \quad (1)$$

Here are a few useful identities implied by Euler's formula.

$$e^{-j\theta} = \cos \theta - j \sin \theta \quad (2a)$$

$$\cos \theta = \operatorname{Re} (e^{j\theta}) \quad (2b)$$

$$= \frac{1}{2} (e^{j\theta} + e^{-j\theta}) \quad (2c)$$

$$\sin \theta = \operatorname{Im} (e^{j\theta}) \quad (2d)$$

$$= \frac{1}{j2} (e^{j\theta} - e^{-j\theta}). \quad (2e)$$

Euler's formula

$$\begin{aligned} e^{-j\theta} + e^{j\theta} &= \cos(-\theta) + j \sin(-\theta) \\ &\quad + \cos(\theta) + j \sin(\theta) \\ &= \cos(\theta) - j \sin(\theta) \\ &\quad + \cos(\theta) + j \sin(\theta) \\ &= 2 \cos(\theta) \end{aligned}$$

## Bibliography

- [1] Tennessee Valley Authority and Tomia. Hydroelectric dam—Wikipedia, The Free Encyclopedia. [Online; accessed 13-February-2018]. 2018.
- [2] Stéfán Baldursson. ?BLDC Motor Modelling and Control - A Matlab®/Simulink® Implementation? mathesis. Chalmers University, 2005.
- [3] Richard C. Booton and Simon Ramo. ?The development of systems engineering? inIEEE Transactions on Aerospace and Electronic Systems: AES-20 (July 1984), pages 306-9.
- [4] William L. Brogan. Modern Control Theory. Third. Prentice Hall, 1991.
- [5] A. Choukchou-Braham and others. Analysis and Control of Underactuated Mechanical Systems. SpringerLink : Bucher. Springer International Publishing, 2013. ISBN: 9783319026367.
- [6] Cameron N. Devine and Rico A.R. Picone. Statum. <https://github.com/CameronDevine/Statum>. 2018.
- [7] P.P.G. Dyke. An Introduction to Laplace Transforms and Fourier Series. 2 edition. Springer Undergraduate Mathematics Series. Springer, 2014. ISBN: 9781447163954.
- [8] H.P. Hsu. Fourier Analysis. Simon & Schuster, 1967. ISBN: 9780671270377.
- [9] John H. Mathews and Russell W. Howell. Complex Analysis for Mathematics and Engineering. 6 edition. Jones and Bartlett Publishers, 2012. ISBN: 9781449604455.
- [10] N.S. Nise. Control Systems Engineering, 7th Edition. Wiley, 2015. ISBN: 9781118800829.
- [11] NOAA. Monthly Average Precipitation 1951-2008 Olympia Regional Airport—NOAA Station. August 2017.
- [12] Jonathan R. Partington. Linear operators and linear systems: An analytical approach to control theory. London Mathematical Society Student Texts. CUP, 2004. ISBN: 9780521546195.
- [13] Derek Rowell and David N. Wormley. System Dynamics: An Introduction. Prentice Hall, 1997.
- [14] S.H. Strogatz and M. Dichter. Nonlinear Dynamics and Chaos. Second. Studies in Nonlinearity. Avalon Publishing, 2016. ISBN: 9780813350844.