trans.char Characteristic transient responses

1 A system's characteristic responses are responses to specific forcing functions—called the singularity functions. The reasons these are "characteristic" are:

characteristic response

singularity functions

1. the singularity functions model commonly interesting system inputs (e.g. a sudden change in the input), and so they can be said to characterize inputs, and
2. the ways in which the system responds to these specific functions reveal aspects of the system (e.g. how quickly it responds), so these responses can be said to characterize systems.

2 Now, one may object that Equation 1b shows that a forcing function needn't look anything like an input due to its being composed of a sum of scaled copies of the input and its derivatives. Yes, but given two key properties of linear, time-invariant (LTI) systems—superposition and the differentiation property—, knowing a system's response y_1 to a forcing function f_1 allows us to construct its response to that input (that is, y_2 for input $u_2 = f_1$) as

superposition

differentiation

$$y_2 = b_m \frac{d^m y_1}{dt^m} + b_{m-1} \frac{d^{m-1} y_1}{dt^{m-1}} + \dots + b_1 \frac{dy_1}{dt} + b_0 y_1$$

I know.

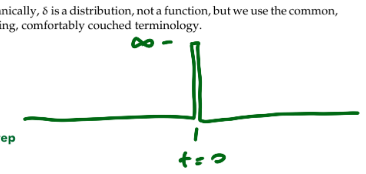
3 There are three singularity functions, which are now defined as piecewise functions of time t.

4 First, the unit impulse or Dirac delta function[1] δ is defined as zero everywhere except at $t = 0$, when it is undefined, and has unity as its integral over all time. When δ is scaled (e.g. 5δ), its integral scales by the same factor. This strange little beast models a sudden "spike" in the input.

unit impulse

Dirac delta

1. Technically, δ is a distribution, not a function, but we use the common, confusing, comfortably couched terminology.



5 Second, the unit step function u_s is defined as zero for $t \leqslant 0$ and unity for $t > 0$. It models a sudden change in the input. Scaling it scales the amount of change. Often, this is considered to be the gold-standard for characterizing the transient response of a system.

unit step

6 Third, the unit ramp function u_r is defined as zero for $t \leqslant 0$ and t for $t > 0$—that is, it is linearly increasing with unity slope. It models a steadily increasing input and is probably the least useful of the singularity functions. Scaling it scales the rate of steady change.

unit ramp