**10.20.** For the rotational system described in Example 9.8, with parameter values $J = 1$ kg-m$^2$, $K = 9$ N-m/rad, and $B = 0.6$ N-s/rad

**(a)** Compute the state transition matrix for the system, based on a time increment of 0.2 s. ✓

**(b)** Compute and plot the solution for (i) the flywheel angular velocity $\Omega_J$ and (ii) the spring torque $T_K$ as a function of time when the motor speed increases suddenly from 0 to 50 rad/s and then remains constant. $u(t) = 50\, u_s(t)$

**(c)** Compute and plot the response of

- **i.** the flywheel speed
- **ii.** the spring torque

as a function of time when the motor speed increases from 0 to 50 rad/s in a linear ramp over a period of 10 s and then remains constant. $5u_r(t) - 5u_r(t-10)$

**(d)** Compare the two solutions with respect to

- **i.** the maximum torque in the spring ✓
- **ii.** the maximum flywheel angular velocity ✓
- **iii.** the steady-state angular velocity and torque

**Example 9.8**

A rotational system consists of an inertial load $J$ mounted in viscous bearings $B$ and driven by an angular velocity source $\Omega_{in}(t)$ through a long light shaft with significant torsional stiffness $K$, as shown in Fig. 9.16. Derive a pair of second-order differential equations for the variables $\Omega_J$ and $\Omega_K$.

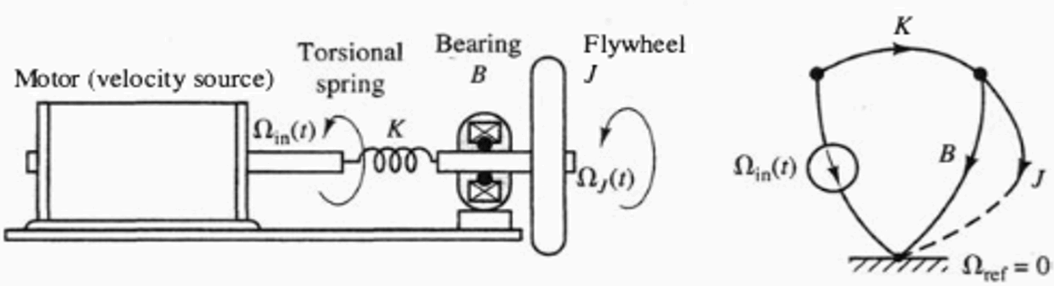


**Figure 9.16:** Rotational system for Example 9.8.

**Solution** The state variables are $\Omega_J$ and $T_K$, and the state equations are

$dt = 0.2$

$$\begin{bmatrix} \dot{\Omega}_j \\ \dot{T}_k \end{bmatrix} = \begin{bmatrix} -B/J & 1/J \\ -K & 0 \end{bmatrix} \begin{bmatrix} \Omega_J \\ T_K \end{bmatrix} + \begin{bmatrix} 0 \\ K \end{bmatrix} \Omega_{in} \tag{i}$$

$$x_{f0}(t) = \int_0^t \Phi(t-\tau) B u(\tau)\, d\tau$$

$$= \sum_{i=0}^{n} \Phi(dt)^n B u(dt\, n)$$

$t = dt\, n$

$n = \frac{t}{dt}$

$\Phi(0.2)$

$\Phi(0.4) = \Phi(0.2)^2$

$\Phi(0) = \Phi(0.2)^0 = I$