RW 10.3

given $\Phi(t)$

a) show $\Phi(T)$ $T=t$ $t=nT$ n integer

$$x(nT) = (\Phi(T))^n x(0)$$

$$\Phi(t_1+t_2) = \Phi(t_1)\Phi(t_2)$$

$$x(t) = \Phi(t)x(0)$$

$$x(nT) = \Phi(nT)x(0)$$

$$= \Phi\left(\sum_{i=0}^{n} T\right)x(0)$$

$$= \left(\prod_{i=0}^{n} \Phi(T)\right)x(0)$$

$$\boxed{= (\Phi(T))^n x(0)}$$

b) if $x_1(0)=2$ $x_2(0)=-1$

$$\Phi(0.1) = \begin{bmatrix} 0.3 & 0.5 \\ 0.0 & 0.4 \end{bmatrix}$$

find
$x(0.1)$
$x(0.3)$

$T = 0.1$

when $t=0.1$ $n=1$

$$x(nT) = (\Phi(T))^n x(0)$$

$$x(0.1) = \begin{bmatrix} 0.3 & 0.5 \\ 0.0 & 0.4 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.3(2)+0.5(-1) \\ -0.4 \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} 0.1 \\ -0.4 \end{bmatrix}}$$

when $t=0.3$ $n=3$

$$x(nT) = (\Phi(T))^n x(0)$$

$$x(0.3) = (\Phi(T))^3 x(0)$$

$$= \begin{bmatrix} 0.3 & 0.5 \\ 0 & 0.4 \end{bmatrix}^3 \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3 & 0.5 \\ 0 & 0.4 \end{bmatrix}\begin{bmatrix} 0.3 & 0.5 \\ 0 & 0.4 \end{bmatrix}\underbrace{\begin{bmatrix} 0.3 & 0.5 \\ 0 & 0.4 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix}}_{x(0.1)}$$

$$= \begin{bmatrix} 0.09 & 0.35 \\ 0 & 0.16 \end{bmatrix}\begin{bmatrix} 0.1 \\ -0.4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.09(0.1) + 0.35(-0.4) \\ 0.16(-0.4) \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} -0.131 \\ 0.064 \end{bmatrix}}$$

c) show that

$$\Phi(T_1+T_2) = \Phi(T_1)\Phi(T_2)$$

$$M\Phi'(T_1+T_2)M^{-1} = M\Phi'(T_1)\cancel{M^{-1}M}\Phi'(T_2)M^{-1}$$

$$\Phi'(T_1+T_2) = \Phi'(T_1)\Phi'(T_2)$$

$$\begin{bmatrix} e^{\lambda_1(T_1+T_2)} & & 0 \\ & e^{\lambda_2(T_1+T_2)} & \\ 0 & & \ddots \end{bmatrix} = \begin{bmatrix} e^{\lambda_1 T_1} & & 0 \\ & e^{\lambda_2 T_1} & \\ 0 & & \ddots \end{bmatrix}\begin{bmatrix} e^{\lambda_1 T_2} & & 0 \\ & e^{\lambda_1 T_2} & \\ 0 & & \ddots \end{bmatrix}$$

$$e^{\lambda_i(T_1+T_2)} = e^{\lambda_i T_1}e^{\lambda_i T_2}$$

$$e^{\lambda_i T_1 + \lambda_i T_2} = e^{\lambda_i T_1}e^{\lambda_i T_2} \quad \square$$