

RW 10.5

$$\Phi(t) = \begin{bmatrix} e^{-2t} & 0 \\ e^{-t} - e^{-2t} & e^{-t} \end{bmatrix} \quad x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

a) find  $\Phi(0.2)$ .

$$\Phi(0.2) = \begin{bmatrix} e^{-2(0.2)} & 0 \\ e^{-0.2} - e^{-2(0.2)} & e^{-0.2} \end{bmatrix}$$

$$= \begin{bmatrix} 0.67 & 0 \\ 0.15 & 0.82 \end{bmatrix}$$

b) find  $x(t)$  in increments of 0.2 using part a with  $0 \leq t \leq 1$

$$x(0.2) = \Phi(0.2) x(0)$$

$$= \begin{bmatrix} 0.67 & 0 \\ 0.15 & 0.82 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.67 \\ 0.97 \end{bmatrix}$$

$$x(0.4) = (\Phi(0.2))^2 x(0)$$

$$= \Phi(0.2) \underbrace{\Phi(0.2) x(0)}_{x(0.2)}$$

$$= \begin{bmatrix} 0.67 & 0 \\ 0.15 & 0.82 \end{bmatrix} \begin{bmatrix} 0.67 \\ 0.97 \end{bmatrix} = \begin{bmatrix} 0.45 \\ 0.9 \end{bmatrix}$$

$$x(0.6) = (\Phi(0.2))^3 x(0) = \Phi(0.2) \underbrace{(\Phi(0.2))^2 x(0)}_{x(0.4)}$$

$$= \begin{bmatrix} 0.67 & 0 \\ 0.15 & 0.82 \end{bmatrix} \begin{bmatrix} 0.45 \\ 0.9 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3 \\ 0.8 \end{bmatrix}$$