15.3. Determine the exponential and sinusoidal forms of the Fourier series and plot the spectrum for the waveform shown in Fig. 15.24. Show how closely the first four terms of the series represents the function.

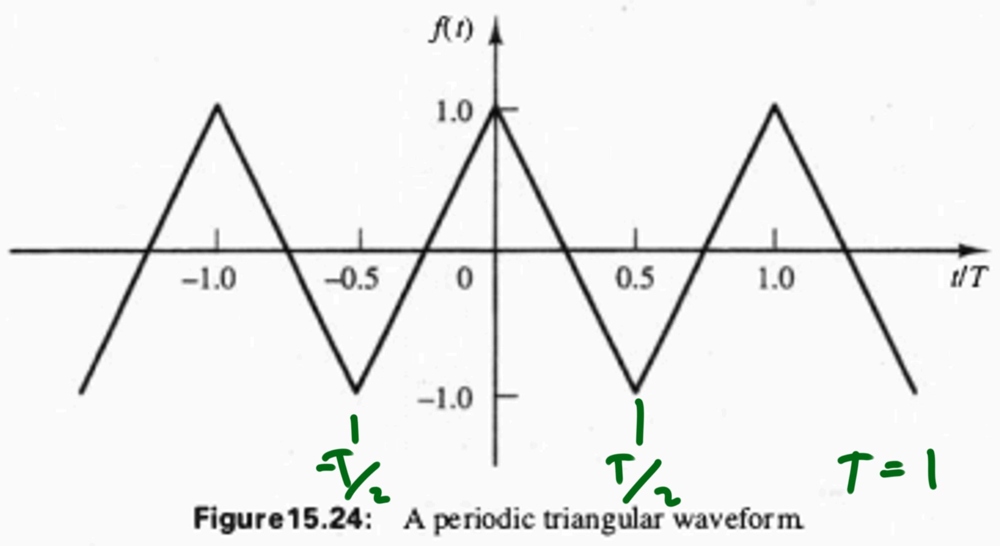

Figure 15.24: A periodic triangular waveform

$b_n = 0$ because even

$$y(t) = \begin{cases} 4t + 1 & t \le 0 \\ -4t + 1 & t > 0 \end{cases}$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} y(t)\cos(w_n t)\,dt$$

$$= \frac{2}{1}\int_{-0.5}^{0} (4t+1)\cos(w_n t)\,dt + \frac{2}{1}\int_{0}^{0.5} (-4t+1)\cos(w_n t)\,dt$$

$$= \frac{2}{1}\left(4\frac{w_n t \sin(w_n t) + \cos(w_n t)}{w_n^2} + \frac{\sin(w_n t)}{w_n}\right)\Bigg|_{-0.5}^{0} + \frac{2}{1}\left(4\frac{\sin(w_n t) - w_n t\cos(w_n t)}{-w_n^2} + \frac{\sin(w_n t)}{w_n}\right)\Bigg|_{0}^{0.5}$$

$$= 2\left(\frac{4}{w_n^2} - 4\frac{w_n(-0.5)\sin(-0.5 w_n) + \cos(-0.5 w_n)}{w_n^2} + \frac{\sin(-0.5 w_n)}{w_n}\right) + 2\left(4\frac{\sin(0.5 w_n) - w_n 0.5\cos(0.5 w_n)}{-w_n^2} + \frac{\sin(0.5 w_n)}{w_n}\right)$$

$$w_n = \frac{2\pi n}{T}$$

$$= 2\left(\frac{\cancel{4}\cancel{T^2}}{\cancel{4}\pi^2 n^2} - \cancel{4}\frac{\frac{2\pi n}{\cancel{T}}(-0.\cancel{5})\sin\left(-0.\cancel{5}\frac{\cancel{2}\pi n}{\cancel{T}}\right) + \cos\left(-0.\cancel{5}\frac{\cancel{2}\pi n}{\cancel{T}}\right)}{\cancel{4}\pi^2 n^2/\cancel{T^2}} + \frac{\sin\left(-0.\cancel{5}\frac{\cancel{2}\pi n}{\cancel{T}}\right)}{2\pi n/\cancel{T}}\right)$$

$$+2\left(\cancel{4}\frac{\sin\left(0.\cancel{5}\frac{\cancel{2}\pi n}{\cancel{T}}\right) - \frac{\cancel{2}\pi n}{\cancel{T}}\cancel{0.5}\cos\left(0.\cancel{5}\frac{\cancel{2}\pi n}{\cancel{T}}\right)}{\cancel{4}\pi^2 n^2/\cancel{T^2}} + \frac{\sin\left(0.5\frac{2\pi n}{\cancel{T}}\right)}{2\pi n/\cancel{T}}\right)$$

$$= 2\left(\frac{1}{n^2\pi^2} - \frac{-\pi n\sin(-\pi n) + \cos(-\pi n)}{\pi^2 n^2} + \frac{\sin(-\pi n)}{2\pi n}\right) + 2\left(\frac{\sin(\pi n) - \pi n\cos(\pi n)}{\pi^2 n^2} + \frac{\sin(-\pi n)}{2\pi n}\right)$$

$$= \frac{2}{\pi^2 n^2} - \frac{2\pi n\sin(\pi n) + 2\cos(\pi n)}{\pi^2 n^2} - \frac{\sin(n\pi)}{\pi n} + \frac{2\sin(\pi n) - 2n\pi\cos(\pi n)}{n^2\pi^2} - \frac{\sin(\pi n)}{\pi n}$$

$$a_n = \frac{2 + 2(1-\pi n)\sin(\pi n) + 2(1-\pi n)\cos(\pi n)}{n^2\pi^2} - \frac{2\sin(n\pi)}{\pi n}$$