

sum.ft Fourier transforms

Table ft.1 is a table with functions of time $f(t)$ on the left and corresponding Fourier transforms $F(\omega)$ on the right. Where applicable, T is the time-domain period, $\omega_0 = 2\pi/T$ is the corresponding angular frequency, $j = \sqrt{-1}$, $a \in \mathbb{R}^+$, and $b, t_0 \in \mathbb{R}$ are constants. Furthermore, f_e and f_o are even and odd functions of time, respectively, and it can be shown that any function f can be written as the sum $f(t) = f_e(t) + f_o(t)$. (H.P. Hsu. Fourier Analysis. Simon & Schuster, 1967. ISBN: 9780671270377)

Table ft.1: Fourier transform identities.

function of time t	function of frequency ω
$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
$f(at)$	$\frac{1}{ a } F(\omega/a)$
$f(-t)$	$F(-\omega)$
$f(t - t_0)$	$F(\omega)e^{-j\omega t_0}$
$f(t) \cos \omega_0 t$	$\frac{1}{2} F(\omega - \omega_0) + \frac{1}{2} F(\omega + \omega_0)$
$f(t) \sin \omega_0 t$	$\frac{1}{2j} F(\omega - \omega_0) - \frac{1}{2j} F(\omega + \omega_0)$
$f_e(t)$	$\text{Re } F(\omega)$
$f_o(t)$	$j \text{Im } F(\omega)$
$F(t)$	$2\pi f(-\omega)$
$f'(t)$	$j\omega F(\omega)$
$\frac{d^n f(t)}{dt^n}$	$(j\omega)^n F(\omega)$
$\int_{-\infty}^t f(\tau) d\tau$	$\frac{1}{j\omega} F(\omega) + \pi f(0) \delta(\omega)$
$-jt f(t)$	$F'(\omega)$
$(-t)^n f(t)$	$\frac{d^n F(\omega)}{d\omega^n}$
$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau$	$F_1(\omega) F_2(\omega)$
$f_1(t) f_2(t)$	$\frac{1}{2\pi} F_1(\omega) * F_2(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(\alpha) F_2(\omega - \alpha) d\alpha$
$e^{-at} u_a(t)$	$\frac{1}{j\omega + a}$
$e^{-a t }$	$\frac{2a}{a^2 + \omega^2}$
e^{-at^2}	$\sqrt{\pi/a} e^{-\omega^2/4a}$
1 for $ t < a/2$, else 0	$\frac{a \sin(\omega a/2)}{\omega a/2}$
$t e^{-at} u_a(t)$	$\frac{1}{(a + j\omega)^2}$
$\frac{t^{n-1}}{(n-1)!} e^{-at} u_a(t)$	$\frac{1}{(a + j\omega)^n}$
$\frac{1}{a^2 + t^2}$	$\frac{\pi}{a} e^{-a \omega }$
$\delta(t)$	1
$\delta(t - t_0)$	$e^{-j\omega t_0}$
$u_a(t)$	$\pi \delta(\omega) + \frac{1}{j\omega}$
$u_a(t - t_0)$	$\pi \delta(\omega) + \frac{1}{j\omega} e^{-j\omega t_0}$
1	$2\pi \delta(\omega)$
t	$2\pi j \delta'(\omega)$
t^n	$2\pi j^n \frac{d^n \delta(\omega)}{d\omega^n}$
$e^{j\omega_0 t}$	$2\pi \delta(\omega - \omega_0)$
$\cos \omega_0 t$	$\pi \delta(\omega - \omega_0) + \pi \delta(\omega + \omega_0)$
$\sin \omega_0 t$	$-j\pi \delta(\omega - \omega_0) + j\pi \delta(\omega + \omega_0)$
$u_a(t) \cos \omega_0 t$	$\frac{j\omega}{\omega_0^2 - \omega^2} + \frac{\pi}{2} \delta(\omega - \omega_0) + \frac{\pi}{2} \delta(\omega + \omega_0)$
$u_a(t) \sin \omega_0 t$	$\frac{\omega_0}{\omega_0^2 - \omega^2} + \frac{\pi}{2j} \delta(\omega - \omega_0) - \frac{\pi}{2j} \delta(\omega + \omega_0)$
$tu_a(t)$	$j\pi \delta'(\omega) - 1/\omega^2$
$1/t$	$\pi j - 2\pi j u_a(\omega)$
$1/t^n$	$\frac{(-j\omega)^{n-1}}{(n-1)!} (\pi j - 2\pi j u_a(\omega))$
$\text{sgn } t$	$\frac{2}{j\omega}$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$

$F(f(t)) = F(\omega)$

$\int_{-\infty}^{\infty} e^{-at} e^{-j\omega t} dt$

$e^{-a(-\infty)} = e^{\infty} = \infty$