

RW 10.12

part c

$$A = \begin{bmatrix} 0 & 1 \\ -25 & 0 \end{bmatrix} \quad \text{find } \Phi(t)$$

find λ :

$$\det(\lambda I - A) = 0$$

$$\left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -25 & 0 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} \lambda & -1 \\ 25 & \lambda \end{vmatrix} = 0$$

$$\lambda^2 + 25 = 0$$

$$\lambda^2 = -25$$

$$\lambda = \sqrt{-25}$$

$$= \pm 5i$$

$$\Lambda = \begin{bmatrix} 5i & 0 \\ 0 & -5i \end{bmatrix}$$

find m :

$$(\lambda_1 I - A) m_1 = 0$$

$$\left(\begin{bmatrix} 5i & 0 \\ 0 & 5i \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -25 & 0 \end{bmatrix} \right) m_1 = \vec{0}$$

$$\begin{bmatrix} 5i & -1 \\ 25 & 5i \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$5i m_{11} - m_{12} = 0$$

$$5i m_{11} = m_{12}$$

$$25 m_{11} + 5i m_{12} = 0$$

$$25 + 5i(5i) = 0$$

$$25 - 25 = 0 \quad \checkmark$$

$$m_1 = \begin{bmatrix} 1 \\ 5i \end{bmatrix}$$

$$(\lambda_2 I - A) m_2 = \vec{0}$$

$$\left(\begin{bmatrix} -5i & 0 \\ 0 & -5i \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -25 & 0 \end{bmatrix} \right) m_2 = \vec{0}$$

$$\begin{bmatrix} -5i & -1 \\ 25 & -5i \end{bmatrix} \begin{bmatrix} m_{21} \\ m_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-5i m_{21} - m_{22} = 0$$

$$-5i m_{21} = m_{22}$$

$$25 m_{21} - 5i m_{22} = 0$$

$$25 - 5i(-5i) = 0$$

$$25 + 25i^2 = 0$$

$$25 - 25 = 0 \quad \checkmark$$

$$m_2 = \begin{bmatrix} 1 \\ -5i \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 1 \\ 5i & -5i \end{bmatrix}$$

find $\Phi'(t)$

$$\Phi'(t) = \begin{bmatrix} e^{5it} & 0 \\ 0 & e^{-5it} \end{bmatrix}$$

find $\Phi(t)$

$$\Phi(t) = M \Phi'(t) M^{-1}$$

$$\det(M) = \begin{vmatrix} 1 & 1 \\ 5i & -5i \end{vmatrix} = -5i(5i) = -25$$

$$M^{-1} = \frac{1}{\det(M)} M^T = \frac{1}{-25} \begin{bmatrix} 1 & 5i \\ 1 & -5i \end{bmatrix}$$

$$\Phi(t) = \begin{bmatrix} 1 & 1 \\ 5i & -5i \end{bmatrix} \begin{bmatrix} e^{5it} & 0 \\ 0 & e^{-5it} \end{bmatrix} \frac{1}{-25} \begin{bmatrix} 1 & 5i \\ 1 & -5i \end{bmatrix}$$

$$= \frac{1}{-25} \begin{bmatrix} e^{5it} & -5it \\ 5ie^{5it} & -5ie^{-5it} \end{bmatrix} \begin{bmatrix} 1 & 5i \\ 1 & -5i \end{bmatrix}$$

$$= \frac{1}{-25} \begin{bmatrix} e^{5it} + e^{-5it} & 5ie^{5it} - 5ie^{-5it} \\ 5ie^{5it} - 5ie^{-5it} & -25e^{5it} - 25e^{-5it} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-e^{5it} - e^{-5it}}{25} & \frac{i(e^{-5it} - e^{5it})}{5} \\ \frac{i(e^{-5it} - e^{5it})}{5} & e^{5it} + e^{-5it} \end{bmatrix}$$

$$x_{fr}(t) = \Phi(t) x(0)$$

$$y_{fr}(t) = B x_{fr}(t) = B \Phi(t) x(0)$$