RW 8.21 (with different input)

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 3u_s(t) \qquad y(0)=1$$

use table second o.1

find w_n $\qquad w_n^2 = 5$

$w_n = \sqrt{5}$ $\qquad \sqrt{5} > 1$

find ζ $\qquad 2\zeta w_n = 2$

$$\zeta = \frac{2}{2w_n} = \frac{\cancel{2}}{\cancel{2}\sqrt{5}} = \frac{1}{\sqrt{5}} < 1$$

$$y_c(t) = \frac{1}{w_n^2}\left(1 - \frac{e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}}\cos(w_d t + \psi)\right)$$

$$= \frac{1}{5}\left(1 - \frac{e^{-t}}{\sqrt{1-1/5}}\cos(w_d + \psi)\right)$$

$$w_d = w_n\sqrt{1-\zeta^2} = \sqrt{5}\sqrt{1-1/5} = \sqrt{\cancel{5}\,\frac{4}{\cancel{5}}} = 2$$

$$\psi = -\operatorname{atan}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right) = -\operatorname{atan}\left(\frac{1/\sqrt{5}}{\sqrt{4/5}}\right) = -\operatorname{atan}\left(\frac{1}{2}\right)$$

$= -0.46$ rad

$$y_c(t) = \frac{1}{5}\left(1 - \frac{\sqrt{5}\,e^{-t}}{2}\cos(2t - 0.46)\right)$$

$$y_{fo}(t) = 3y_c(t)$$

$$y_{fr}(t) = y(0)\frac{e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}}\cos(w_d t + \psi)$$

$$= \frac{\sqrt{5}\,e^{-t}}{2}\cos(2t - 0.46)$$

$$y(t) = y_{fo}(t) + y_{fr}(t)$$

$$= \frac{3}{5}\left(1 - \frac{\sqrt{5}\,e^{-t}}{2}\cos(2t-0.46)\right) + \frac{\sqrt{5}\,e^{-t}}{2}\cos(2t-0.46)$$

$$= \frac{3}{5} + \left(1 - \frac{3}{5}\right)\frac{\sqrt{5}\,e^{-t}}{2}\cos(2t-0.46)$$

$$= \frac{3}{5} + \frac{\cancel{2}}{5}\frac{\sqrt{5}\,e^{-t}}{\cancel{2}}\cos(2t-0.46)$$

$$= \frac{3}{5} + \frac{e^{-t}}{\sqrt{5}}\cos(2t-0.46)$$