ME 370 – System Dynamics and Control

Midterm Exam 1

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Contents

Directions: take-home, open notes, open book. Use your own paper, work neatly, and clearly mark your answers. MATLAB and other programming languages may not be used. Partial credit may be given. Submit as a single pdf.

For 5 points of extra credit, participate in this study on why students choose to study STEM related subjects before data collection closes on February 18th.

`https://tinyurl.com/ThesisSTEMSurvey`

Problem canada

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Given a differential equation,

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 25y = f(t),$$

with initial conditions $\frac{dy}{dt}\big|_{t=0} = 0$ and $y(0) = 8$, find

a the undamped natural frequency ω_n and damping ratio ζ,
b the free response $y_{fr}(t)$,
c the forced response due to a Dirac delta forcing function $f(t) = \delta(t)$,
d the forced response due to a unit step forcing function $f(t) = u_s(t)$,
e the forced response due to a unit ramp forcing function $f(t) = r(t)$,
f the forced response to the forcing function,

$$f(t) = 7\delta(t) - 4u_s(t) + 6r(t),$$

and
g the total response from the initial condition and the forcing function in part **f**.

Problem argentina

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Given a state space system,

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du,$$

with,

$$A = \begin{bmatrix} -8 & -6 \\ 3 & 1 \end{bmatrix} \text{ and}$$
$$C = \begin{bmatrix} 2 & -1 \end{bmatrix},$$

find

✓ **a** the system's Eigen values λ_i,
✓ **b** the Eigen vectors m_i and modal matrix M,
✓ **c** the diagonalized state transition matrix $\Phi'(t)$,
✓ **d** the state transition matrix in the original basis $\Phi(t)$, and
✓ **e** the output free response $y_{fr}(t)$ due to an initial condition $x(0) = [4, -1]^T$.

$$\det(\lambda I - A) = 0$$

$$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -8 & -6 \\ 3 & 1 \end{bmatrix}\right) = 0$$

$$\begin{vmatrix} \lambda+8 & 6 \\ -3 & \lambda-1 \end{vmatrix} = 0$$

$$(\lambda+8)(\lambda-1) + 18 = 0$$

$$\lambda^2 + 8\lambda - \lambda - 8 + 18 = 0$$

$$\lambda^2 + 7\lambda + 10 = 0$$

$$(\lambda+2)(\lambda+5) = 0$$

$$\boxed{\lambda = -2, -5}$$

$$(\lambda_1 I - A)m_1 = 0$$

$$\left(\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} -8 & -6 \\ 3 & 1 \end{bmatrix}\right) m_1 = 0$$

$$\begin{bmatrix} 6 & 6 \\ -3 & -3 \end{bmatrix}\begin{bmatrix} m_{11} \\ m_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$6m_{11} + 6m_{21} = 0$$
$$m_{11} + m_{21} = 0$$
$$m_{11} = -m_{21}$$

$$\boxed{m_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

$$(\lambda_2 I - A)m_2 = 0$$

$$\left(\begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix} - \begin{bmatrix} -8 & -6 \\ 3 & 1 \end{bmatrix}\right) m_2 = 0$$

$$\begin{bmatrix} 3 & 6 \\ -3 & -6 \end{bmatrix}\begin{bmatrix} m_{12} \\ m_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$3m_{12} + 6m_{22} = 0$$
$$m_{12} + 2m_{22} = 0$$
$$m_{12} = -2m_{22}$$

$$\boxed{m_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}}$$

$$\boxed{M = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}}$$

$$\boxed{\Phi'(t) = \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-5t} \end{bmatrix}}$$

$$\Phi(t) = M\,\Phi'(t)\,M^{-1}$$

$$\det(M) = \begin{vmatrix} 1 & 2 \\ -1 & -1 \end{vmatrix} = -1 + 2 = 1$$

$$= \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-5t} \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$M^{-1} = \frac{1}{\det(M)}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} e^{-2t} & 2e^{-5t} \\ -e^{-2t} & -e^{-5t} \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 2e^{-5t} - e^{-2t} & 2e^{-5t} - 2e^{-2t} \\ e^{-2t} - e^{-5t} & 2e^{-2t} - e^{-5t} \end{bmatrix}}$$

$$y_{fr}(t) = Cx + \cancel{Du} = Cx_{fr}(t) = C\,\Phi(t)\,x(0)$$

$$= \begin{bmatrix} 2 & -1 \end{bmatrix}\begin{bmatrix} 2e^{-5t} - e^{-2t} & 2e^{-5t} - 2e^{-2t} \\ e^{-2t} - e^{-5t} & 2e^{-2t} - e^{-5t} \end{bmatrix}\begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \end{bmatrix}\begin{bmatrix} 6e^{-5t} - 2e^{-2t} \\ 2e^{-2t} - 3e^{-5t} \end{bmatrix} = \boxed{15e^{-5t} - 6e^{-2t}}$$

Problem morocco

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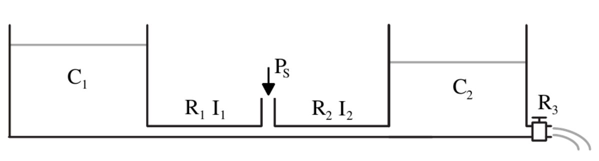
For the system below with a pressure source P_s, fluid resistances R_i, fluid inertances I_i, and fluid capacitances C_i, find

a the linear graph,
b the normal tree, and
c the system state variables and system order.



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Problem taiwan

The cooling system for a desktop computer CPU is shown below. We can consider the CPU as a heat flow source Q_s. A thermal interface material is then used to transfer heat from the CPU to the cooling system. However, this thermal interface material is not a perfect thermal conductor. The cooling system then consists of a base plate with thermal capacitance and a "heat pipe" which moves the heat away from the CPU. The "heat pipe" is again an imperfect thermal conductor. At the other end of the "heat pipe" is a constant temperate (which we can model as a temperature source). For this system, find

a the linear graph,
b the normal tree, and
c the system state variables and system order.