freq.bode Bode plots

1 This lecture also appears in Control: an introduction.

2 Given Eq. 10, we are often most-interested in the magnitude $|H(j\omega)|$ and phase $\angle H(j\omega)$ of the frequency response function. Each of these is a function of angular frequency ω, so plotting $|H(j\omega)|$ vs. ω and $\angle H(j\omega)$ vs. ω is quite useful. Bode plots are such plots with axes scaled in a specific manner.

Bode plots

3 A Bode plot is a useful graphical representation of the frequency response of a system. Let $|U(\omega)|$ and $|Y(\omega)|$ be the complex amplitudes of the input and the output, respectively. Recall that the magnitude of the frequency response function $|H(j\omega)|$ can be expressed as

$$H(s) = \frac{Y(s)}{U(s)}$$

$$H(j\omega) = \frac{Y(\omega)}{U(\omega)}$$

Equation 1 frequency response function as an amplitude ratio

$$|H(j\omega)| = \left|\frac{Y(\omega)}{U(\omega)}\right|$$

4 This is a ratio of amplitudes, and so it is akin to amplitude ratios commonly expressed in decibels (dB). However, the magnitude ratio of Eq. 1 is not dimensionless, and therefore cannot be expressed as decibel in the strict sense. Nevertheless, it is standard usage in system dynamics and control theory use the familiar formula to compute the logarithmic magnitude

decibels

Equation 2 logarithmic magnitude of $H(j\omega)$ in "dB"

$$|H(j\omega)| = 20 \log_{10} |H(j\omega)| \text{ dB}$$

5 The phase is usually plotted in degrees, and the ω-axis is logarithmic in both plots. The two plots are typically tiled vertically with the magnitude plot above the phase. We now work a simple example.

Example freq.bode-1 **re: A simple Bode plot**

Let a system have transfer function $H(s) = s$, a single zero at the origin. Find the frequency response function and draw the Bode plot for the system.

$$H(s) = \frac{s}{1}$$

$$H(j\omega) = H(s)\big|_{s \to j\omega}$$
$$= j\omega$$

$$|H(j\omega)| = \sqrt{R(H(j\omega))^2 + Im(H(j\omega))^2} = \sqrt{0 + \omega^2} = \omega$$

$$\angle H(j\omega) = \tan^{-1}\left(\frac{Im(H(j\omega))}{Re(H(j\omega))}\right) = \tan^{-1}\left(\frac{\omega}{0}\right) = 90^\circ$$

$20 \log_{10} \omega$

ω	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}
dB	-40	-20	0	20	40