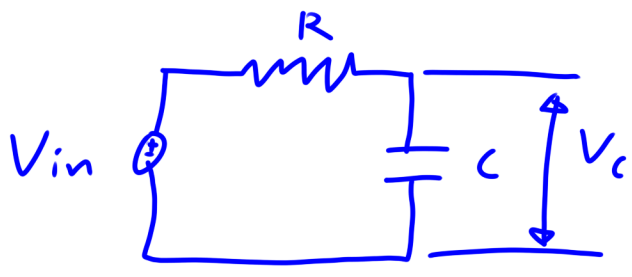
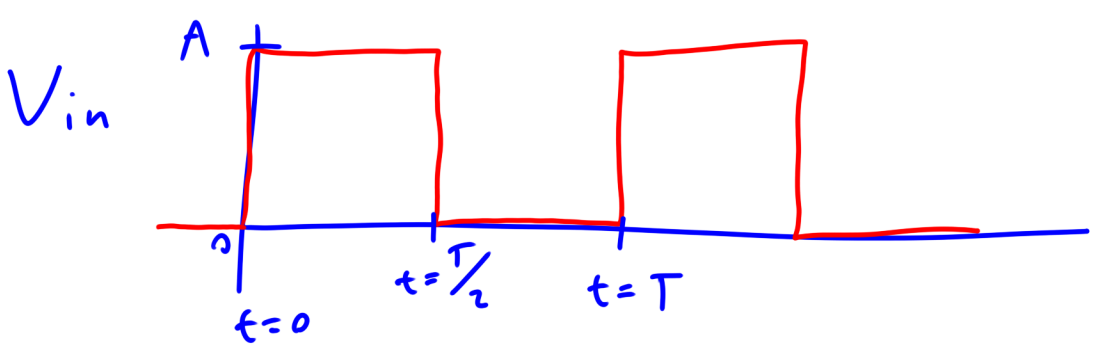
RC circuit

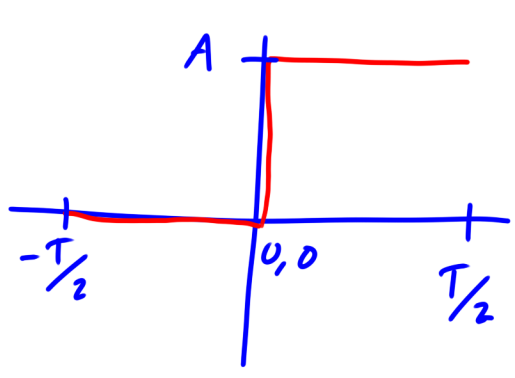


$$H(s) = \frac{1}{1+RCs} = \frac{V_c(s)}{V_{in}(s)}$$



$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} V_{in}(t)\cos(\omega_n t)\,dt$$

$$= \frac{2}{T}\int_{0}^{T/2} A\cos(\omega_n t)\,dt$$



$$= \frac{2A}{T\omega_n}\sin(\omega_n t)\Big|_0^{T/2} = \frac{2A}{T\omega_n}\sin\left(\omega_n \frac{T}{2}\right)$$

$$\omega_n = \frac{2\pi n}{T} \qquad = \frac{2A}{T\omega_n}\sin\left(\frac{2\pi n}{T}\frac{T}{2}\right) = 0$$

$$a_0 = \frac{2}{T}\int_{-T/2}^{T/2} V_{in}(t)\,dt = \frac{2}{T}\int_0^{T/2} A\,dt = \frac{2A}{T}t\Big|_0^{T/2}$$

$$= \frac{2A}{T}\frac{T}{2} = A$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} V_{in}(t)\sin(\omega_n t)\,dt = \frac{2}{T}\int_0^{T/2} A\sin(\omega_n t)\,dt$$

$$= \frac{-2A}{T\omega_n}\cos(\omega_n t)\Big|_0^{T/2} = \frac{-2A}{T\omega_n}\cos\left(\omega_n \frac{T}{2}\right) + \frac{2A}{T\omega_n}$$

$$= \frac{-2A}{T\frac{2\pi n}{T}}\cos\left(\frac{2\pi n}{T}\frac{T}{2}\right) + \frac{2A}{T\frac{2\pi n}{T}}$$

$$= \frac{-A}{\pi n}\cos(n\pi) + \frac{A}{\pi n}$$

$$= \frac{A}{n\pi}\left(1-\cos(n\pi)\right)$$

$$V_{in}(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} b_n \sin(\omega_n t) \checkmark$$

$$H(j\omega) = H(s)\Big|_{s\to j\omega} = \frac{1}{1+RCs}\Big|_{s\to j\omega} = \frac{1}{1+RCj\omega}\frac{1-RCj\omega}{1-RCj\omega}$$

$$= \frac{1-RCj\omega}{1+(RC\omega)^2} \qquad Re(H(j\omega)) = \frac{1}{1+(RC\omega)^2}$$

$$Im(H(j\omega)) = \frac{-RC\omega}{1+(RC\omega)^2}$$

$$|H(j\omega)| = \sqrt{Re(H(j\omega))^2 + Im(H(j\omega))^2}$$

$$= \sqrt{\left(\frac{1}{1+(RC\omega)^2}\right)^2 + \left(\frac{-RC\omega}{1+(RC\omega)^2}\right)^2}$$

$$= \frac{\sqrt{1+(RC\omega)^2}}{1+(RC\omega)^2} = \frac{1}{\sqrt{1+(RC\omega)^2}}$$

$$\angle H(j\omega) = \tan^{-1}\left(\frac{Im(H(j\omega))}{Re(H(j\omega))}\right)$$

$$= \tan^{-1}\left(\frac{\frac{-RC\omega}{1+(RC\omega)^2}}{\frac{1}{1+(RC\omega)^2}}\right) = \tan^{-1}(-RC\omega)$$

$$V_c(t) = \frac{a_0}{2}|H(j0)| + \sum_{n=1}^{\infty} b_n |H(j\omega_n)| \sin(\omega_n t + \angle H(j\omega_n))$$