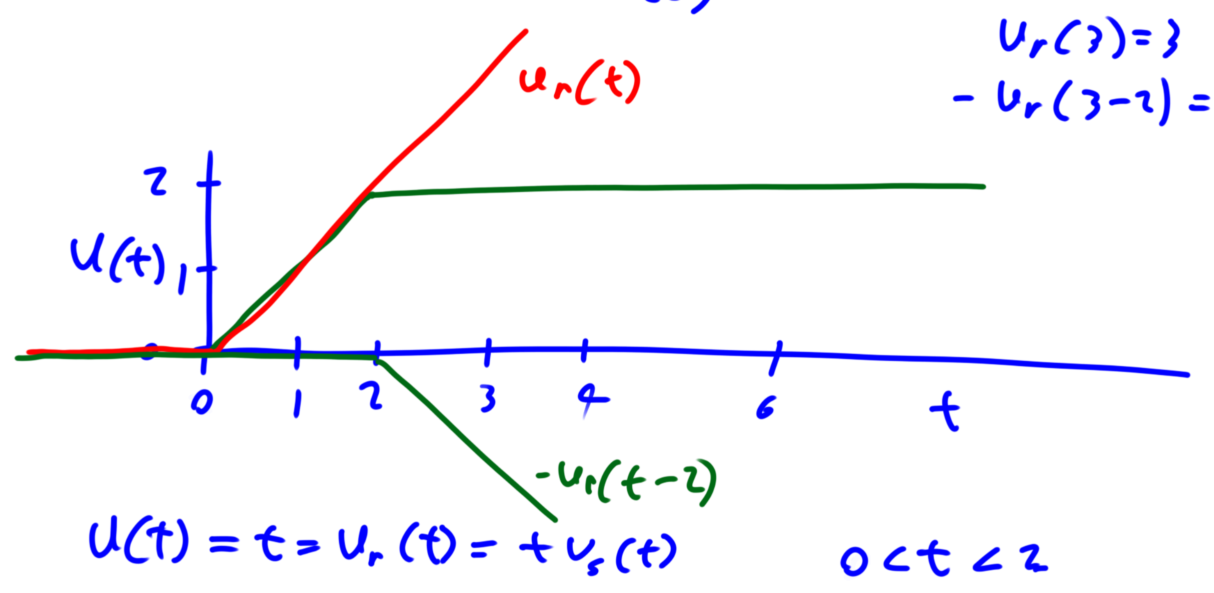
RW 15.26

$$H(s) = \frac{4}{2s+1} = \frac{Y(s)}{U(s)}$$

$U_r(3) = 3$

$-U_r(3-2) = -1$



$U(t) = t = U_r(t) = t\,u_s(t) \qquad 0 < t < 2$

$$Y(s) = H(s)U(s) \qquad U(s) = \mathcal{L}(u(t)) = \frac{1}{s^2}$$

$$= \frac{4}{2s+1}\,\frac{1}{s^2}$$

$$= \frac{16}{2s+1} - \frac{8}{s} + \frac{4}{s^2}$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{16}{2s+1} - \frac{8}{s} + \frac{4}{s^2}\right) = \mathcal{L}^{-1}\left(\frac{16}{2s+1}\right) - \mathcal{L}^{-1}\left(\frac{8}{s}\right) + \mathcal{L}^{-1}\left(\frac{4}{s^2}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{8}{s+1/2}\right) - \mathcal{L}^{-1}\left(\frac{8}{s}\right) + \mathcal{L}^{-1}\left(\frac{4}{s^2}\right)$$

$$= 8e^{-t/2} - 8u_s(t) + 4U_r(t) = 8e^{-t/2} - 8 + 4t$$

$$y(2) = 8e^{-2/2} \cancel{-8+4(2)} = 8e^{-1} = \underline{2.94}$$

$t > 2$

$$u(t) = 2 = 2u_s(t) \qquad \mathcal{L}(u(t)) = \frac{2}{s} = U(s)$$

$$H(s) = \frac{4}{2s+1} = \frac{Y(s)}{U(s)}$$

$$(2s+1)Y(s) = 4U(s)$$

$$2\frac{dy}{dt} + y = 4u(t)$$

$$2sY(s) - y(2) + Y(s) = 4U(s)$$

$$(2s+1)Y(s) - y(2) = 4U(s)$$

$$(2s+1)Y(s) = 4U(s) + y(2)$$

$$Y(s) = \frac{4U(s) + y(2)}{2s+1}$$

$$= \frac{4}{2s+1}\,\frac{2}{s} + \frac{y(2)}{2s+1}$$

$$= \frac{8}{s} - \frac{16}{2s+1} + \frac{2.94}{2s+1}$$

$$= \frac{8}{s} - \frac{13.06}{2s+1} = \frac{8}{s} - \frac{6.53}{s+1/2}$$

$$y(t) = \mathcal{L}^{-1}(Y(s))$$

$$= 8u_s(t) - 6.53e^{-t/2}$$

Method 2

$$\mathcal{L}(u(t)) = \int_0^\infty u(t)e^{-st}\,dt$$

$$= \int_0^2 t e^{-st}\,dt + \int_2^\infty 2e^{-st}\,dt$$

$$= \frac{-e^{-st}(st+1)}{s^2}\Bigg|_0^2 + \frac{-2e^{-st}}{s}\Bigg|_2^\infty$$

$$= \frac{-e^{-2s}(2s+1)}{s^2} - \frac{-1}{s^2} + \lim_{t\to\infty}\cancel{\frac{-2e^{-st}}{s}} - \frac{-2e^{-2s}}{s}$$

(→ 0)

$$= \cancel{\frac{-2}{s}e^{-2s}} - \frac{e^{-2s}}{s^2} + \frac{1}{s^2} + \cancel{\frac{2}{s}e^{-2s}}$$

$$= \underline{\frac{1}{s^2} - \frac{1}{s^2}e^{-2s}} = U(s)$$

$$u(t) = \mathcal{L}^{-1}(U(s)) = \underline{U_r(t) - U_r(t-2)}$$

$$Y(s) = H(s)U(s)$$

$$= \frac{4}{2s+1}\left(\frac{1}{s^2} - \frac{1}{s^2}e^{-2s}\right)$$

$$= -\frac{4e^{-2s} - 4}{s^2} - \frac{8e^{-2s} - 8}{s} - \frac{16e^{-2s} - 16}{2s+1}$$

$$= \frac{-4}{s^2}e^{-2s} + \frac{4}{s^2} - \frac{8}{s}e^{-2s} + \frac{8}{s} - \frac{8}{s+1/2}e^{-2s} + \frac{8}{s+1/2}$$

$$= -4u_r(t-2) + U_r(t) - 8u_s(t-2) + 8u_s(t)$$
$$- 8e^{\frac{-(t-2)}{2}}u_s(t-2) + 8e^{-t/2}$$