imp.divide The divider method

1 In Electronics, we developed the useful voltage divider formula for quickly analyzing how voltage divides among series electronic impedances. This can be considered a special case of a more general across-variable divider equation for any elements described by an impedance. After developing the across-variable divider, we also introduce the through-variable divider, which divides an input through-variable among parallel elements.

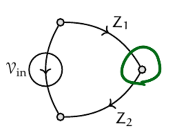


Figure divide.1: the two-element across-variable divider.

across−variable divider

Across-variable dividers

2 First, we develop the solution for the two-element across-variable divider shown in Figure divide.1. We choose the across-variable across Z_2 as the output. The analysis follows the impedance method of Lecture imp.tf, solving for $\mathcal{V}_2$.

1. Derive four independent equations.
   a) The normal tree is chosen to consist of $\mathcal{V}_{in}$ and Z_2.
   b) The elemental equations are

   $$V_1 = F_1 Z_1$$
   $$V_2 = F_2 Z_2$$

   c) The continuity equation is $F_2 = F_1$
   d) The compatibility equation is
   $$V_1 = V_{in} - V_2$$
2. Solve for the output $\mathcal{V}_2$. From the elemental equation for Z_2,

$$\begin{aligned} \mathcal{V}_2 &= \mathcal{F}_\infty Z_2 \\ &= \frac{\mathcal{V}_1}{Z_1} Z_2 \\ &= \frac{Z_2}{Z_1}(\mathcal{V}_{in} - \mathcal{V}_2) \quad \Rightarrow \\ \mathcal{V}_2 &= \frac{Z_2}{Z_1 + Z_2}\mathcal{V}_{in}. \end{aligned}$$

3 A similar analysis can be conducted for n impedance elements.

Equation 1 general across−variable divider

For the output across Z_K in series with n impedances with input V_{in} is

$$V_K = \frac{Z_K}{Z_1 + Z_2 + \cdots + Z_K + \cdots + Z_n} V_{in}$$

Through-variable dividers

4 By a similar process, we can analyze a network that divides a through-variable into n parallel impedance elements.

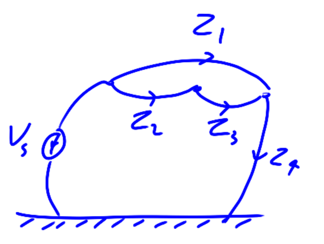
Equation 2 general through−variable divider

For the output through variable through Z_K in parallel with n impedances with input F_{in}

$$F_K = \frac{1/Z_K}{1/Z_1 + \cdots + 1/Z_K + \cdots + 1/Z_n} F_{in}$$

Transfer functions using dividers

5 An excellent shortcut to deriving a transfer function is to use the across- and through-variable divider rules instead of solving the system of algebraic equations, as in Lec. imp.tf. An algorithm for this process is as follows.

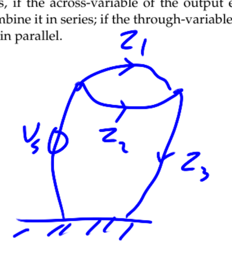


$$\frac{V_3(s)}{V_s(s)}$$

cannot use across variable divider

1. Identify the element associated with an output variable Y_i. Call it the output element.
2. Identify the source associated with an input variable U_j. Set all other sources to zero.
3. Transform the network to be an across- or through-variable divider that includes the "bare" (uncombined) output element's output variable.[6]
   a) If necessary, form equivalent impedances of portions of the network, being sure to leave the output element's output variable alone.
   b) If necessary, transform the source à la Norton or Thévenin.
4. Apply the across- or through-variable divider equation.
5. If necessary, use the elemental equation of the output element to trade output across- and through-variables.
6. If necessary, use the source transformation equation of the input to trade input across- and through-variables.
7. Divide both sides by the input variable.

6. In other words, if the across-variable of the output element is the output, do not combine it in series; if the through-variable is the output, do not combine it in parallel.



$$\frac{V_3(s)}{V_s(s)} = H(s)$$

$$F_3(s) Z_3 = V_3(s)$$

$$\frac{F_3(s)}{V_s(s)} = \frac{1}{Z_3} H(s) \qquad \frac{F_3(s)}{V_3(s)} = \frac{1}{Z_3}$$

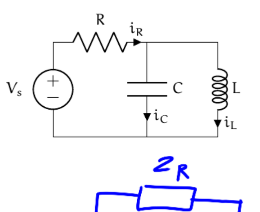
$$\frac{F_3(s)}{V_3(s)} \quad \frac{V_3(s)}{V_s(s)}$$

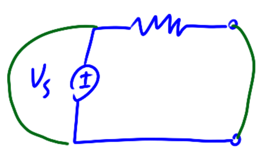
6 It turns out that, despite its many "if necessary" clauses, very often this "shortcut" is easier than the method of Lecture imp.tf for low-order systems if only a few transfer functions are of interest.

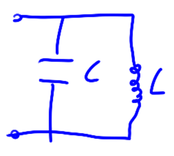
Example imp.divide−1 **re: a circuit transfer function using a divider**

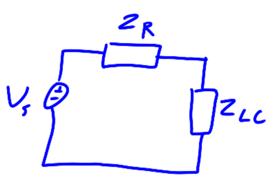
Given the circuit shown with voltage source V_s and output v_L,

a. what is the transfer function $\frac{V_L}{V_s}$?
b. Without transforming the source, find the transfer function $\frac{I_L}{V_s}$.
c. Transforming the source, find $\frac{I_L}{V_s}$.









$$V_L = \frac{Z_{LC}}{Z_R + Z_{LC}} V_s \qquad Z_{LC} = \frac{1}{\frac{1}{Z_L} + \frac{1}{Z_C}} = \frac{1}{\frac{1}{Ls} + Cs} \frac{Ls}{Ls}$$

$$\frac{V_L}{V_s} = \frac{Z_{LC}}{Z_R + Z_{LC}} = \frac{\frac{Ls}{1 + LCs^2}}{R + \frac{Ls}{1 + LCs^2}} \frac{1 + LCs^2}{1 + LCs^2} \qquad = \frac{Ls}{1 + LCs^2}$$

$$\boxed{\frac{V_L}{V_s} = \frac{Ls + L^2Cs^3}{R + Ls + L^2Cs^3}}$$

$$V_L = \frac{Z_{LC}}{Z_R + Z_{LC}} V_s \qquad I_L Z_L = V_L$$

$$I_L Z_L = \frac{Z_{LC}}{Z_R + Z_{LC}} V_s$$

$$I_L = \frac{1}{Z_L} \frac{Z_{LC}}{Z_R + Z_{LC}} V_s$$

$$= \frac{1}{Ls} \frac{Ls + L^2Cs^3}{R + Ls + L^2Cs^3} V_s$$

$$= \frac{1 + LCs^2}{R + Ls + L^2Cs^3} V_s$$

$$\boxed{\frac{I_L}{V_s} = \frac{1 + LCs^2}{R + Ls + L^2Cs^3}}$$

$$Z_e = Z_R$$

$$I_e = V_s Z_R$$

$$I_L = \frac{1/Z_L}{1/Z_e + 1/Z_C + 1/Z_L} I_e$$

$$= \frac{1/Z_L}{1/Z_e + 1/Z_C + 1/Z_L} Z_R V_s$$

$$= \frac{1/Ls}{1/R + Cs + 1/Ls} R V_s$$

$$= \frac{R}{Ls/R + LCs^2 + 1} V_s$$

$$= \frac{R^2}{Ls + LCRs^2 + R} V_s$$

$$\boxed{\frac{I_L}{V_s} = \frac{R^2}{Ls + RCLs^2 + R}}$$