



fourier series

$$T=2$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} -t \cos(\omega_n t) dt$$

$$= - \int_{-1}^1 t \cos(\omega_n t) dt$$

$$\int u dv = uv - \int v du$$

$$u = t \quad dv = \cos(\omega_n t) dt$$

$$du = dt \quad v = \frac{\sin(\omega_n t)}{\omega_n}$$

$$= - \left( t \frac{\sin(\omega_n t)}{\omega_n} - \int_{-1}^1 \frac{\sin(\omega_n t)}{\omega_n} dt \right)$$

$$= - \left( t \frac{\sin(\omega_n t)}{\omega_n} + \frac{\cos(\omega_n t)}{\omega_n^2} \right) \Big|_{-1}^1$$

$$= - \left( \frac{\sin(\omega_n)}{\omega_n} + \frac{\cos(\omega_n)}{\omega_n^2} + \frac{\sin(-\omega_n)}{\omega_n} - \frac{\cos(-\omega_n)}{\omega_n^2} \right)$$

$$= 0 \frac{2 \cos(\omega_n)}{\omega_n^2}$$

$$b_n = - \int_{-1}^1 t \sin(\omega_n t) dt$$

$$u = t \quad dv = \sin(\omega_n t) dt$$

$$du = dt \quad v = -\frac{\cos(\omega_n t)}{\omega_n}$$

$$= - \left( \frac{-t \cos(\omega_n t)}{\omega_n} - \int_{-1}^1 \frac{-\cos(\omega_n t)}{\omega_n} dt \right)$$

$$= \frac{t \cos(\omega_n t)}{\omega_n} - \frac{\sin(\omega_n t)}{\omega_n^2} \Big|_{-1}^1$$

$$= \frac{\cos(\omega_n)}{\omega_n} - \frac{\sin(\omega_n)}{\omega_n^2} + \frac{\cos(-\omega_n)}{\omega_n} + \frac{\sin(-\omega_n)}{\omega_n^2}$$

$$= \frac{2 \cos(\omega_n)}{\omega_n} - \frac{2 \sin(\omega_n)}{\omega_n^2}$$