ME 370 – System Dynamics and Control

Midterm Exam 2

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Contents

Directions: take-home, open notes, open book. Use your own paper, work neatly, and clearly mark your answers. MATLAB and other programming languages may not be used. Partial credit may be given. Submit as a single PDF file.

Problem maximization

_____/10 p.

List three ways that a system's transfer function can be determined.

linear graph → SS → TF

linear graph → Impedance Methods → TF

algebra with elemental eq'ns → IO ODE → TF

impulse response → TF

Problem supremum

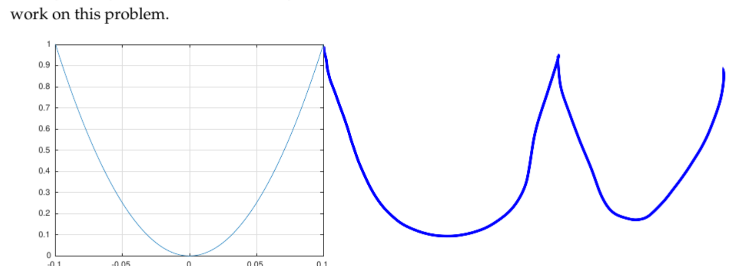
_____/25 p.

Given a periodic function $f(t) = 100t^2$ with a period $T = 0.2$ centered about the origin, also shown below:

a calculate the first 5 components of the fourier series, and

b write the first 5 terms of the fourier series representation of the function.

Note: you may use MATLAB to check your work on this problem.



$$a_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\cos(\omega_n t)\,dt$$

$$= \frac{1}{0.2}\int_{-0.1}^{0.1} 100t^2\cos(\omega_n t)\,dt$$

$$= 500\int_{-0.1}^{0.1} t^2\cos(\omega_n t)\,dt$$

$u = t^2$ $\quad dv = \cos(\omega_n t)\,dt$

$du = 2t\,dt$ $\quad v = \frac{\sin(\omega_n t)}{\omega_n}$

$$= 500\left(t^2\frac{\sin(\omega_n t)}{\omega_n} - \int_{-0.1}^{0.1}\frac{\sin(\omega_n t)}{\omega_n}2t\,dt\right)$$

$$= 500\left(t^2\frac{\sin(\omega_n t)}{\omega_n} - \frac{2}{\omega_n}\int_{-0.1}^{0.1} t\sin(\omega_n t)\,dt\right)$$

$u = t$ $\quad dv = \sin(\omega_n t)\,dt$

$du = dv$ $\quad v = \frac{-\cos(\omega_n t)}{\omega_n}$

$$= 500\left(t^2\frac{\sin(\omega_n t)}{\omega_n} - \frac{2}{\omega_n}\left(t\frac{-\cos(\omega_n t)}{\omega_n} - \int_{-0.1}^{0.1}\frac{-\cos(\omega_n t)}{\omega_n}\,dt\right)\right)$$

$$= 500\left(t^2\frac{\sin(\omega_n t)}{\omega_n} - \frac{2}{\omega_n}\left(t\frac{-\cos(\omega_n t)}{\omega_n} - \frac{-\sin(\omega_n t)}{\omega_n^2}\right)\right)\Bigg|_{-0.1}^{0.1}$$

$$= 500\left(\frac{1}{100}\frac{\sin(\omega_n/10)}{\omega_n} - \frac{2}{\omega_n}\left(\frac{-\cos(\omega_n/10)}{10\omega_n} + \frac{\sin(\omega_n/10)}{\omega_n^2}\right)\right)$$

$$- 500\left(\frac{1}{100}\frac{\sin(-\omega_n/10)}{\omega_n} - \frac{2}{\omega_n}\left(\frac{\cos(-\omega_n/10)}{10\omega_n} + \frac{\sin(-\omega_n/10)}{\omega_n^2}\right)\right)$$

$$= 1000\left(\frac{1}{100}\frac{\sin(\omega_n/10)}{\omega_n} - \frac{2}{\omega_n}\left(\frac{-\cos(\omega_n/10)}{10\omega_n} + \frac{\sin(\omega_n/10)}{\omega_n^2}\right)\right)$$

$$a_n = \left(\frac{10}{\omega_n} - \frac{2000}{\omega_n^3}\right)\sin(\omega_n/10) + \frac{200}{\omega_n^2}\cos(\omega_n/10)$$

$$\omega_n = \frac{2\pi n}{T} = 10\pi n$$

$$a_n = \cancel{\left(\frac{1}{\pi n} - \frac{2}{\pi^3 n^3}\right)\sin(\pi n)} + \frac{2}{\pi^2 n^2}\cos(\pi n)$$

$$a_n = \frac{2}{\pi^2 n^2}\cos(n\pi) \qquad b_n = 0$$

$$a_1 = \frac{-2}{\pi^2} \qquad a_2 = \frac{1}{2\pi^2} \qquad a_3 = \frac{-2}{9\pi^2}$$

$$a_4 = \frac{1}{8\pi^2} \qquad a_5 = \frac{-2}{25\pi^2}$$

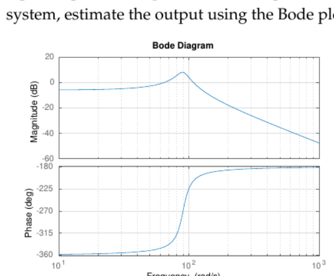
$\omega_1 = 10\pi$ $\quad \omega_2 = 20\pi$ $\quad \omega_3 = 30\pi$ $\quad \omega_4 = 40\pi$ $\quad \omega_5 = 50\pi$

$$f(t) \approx \frac{-2}{\pi^2}\cos(10\pi t) + \frac{1}{2\pi^2}\cos(20\pi t) + \frac{-2}{9\pi^2}\cos(30\pi t) + \frac{1}{8\pi^2}\cos(40\pi t) + \frac{-2}{25\pi^2}\cos(50\pi t)$$

Problem bigish

_____/15 p.

The Bode plot for a system is shown below. Using the Fourier series representation of the signal in problem supremum as an input to this system, estimate the output using the Bode plot.



$\angle H(j31) = -350° = -6.1$

$\angle H(j63) = -345° = -6.0$

$\angle H(j94) = -235° = -4.2$

$\angle H(j126) = -200° = -3.5$

$\angle H(j157) = -195° = -3.4$

$|H(j31)| = -4\text{ dB} = 0.6$

$|H(j63)| = 0\text{ dB} = 1$

$|H(j94)| = 6\text{ dB} = 2$

$|H(j126)| = -7\text{ dB} = 0.4$

$|H(j157)| = -10\text{ dB} = 0.3$

$\text{dB} = 20\log_{10}\text{mag}$

$\text{mag} = 10^{\frac{\text{dB}}{20}}$

$$0.6\,\frac{-2}{\pi^2}\cos(10\pi t - 6.1) + \frac{1}{2\pi^2}\cos(20\pi t - 6) + 2\,\frac{-2}{9\pi^2}\cos(30\pi t - 4.2) + 0.4\,\frac{1}{8\pi^2}\cos(40\pi t - 3.5) + 0.3\,\frac{-2}{25\pi^2}\cos(50\pi t - 3.4)$$

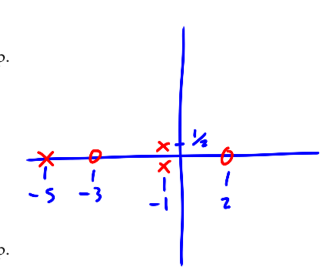
Problem enormous

_____/15 p.

Given a transfer function,

$$H(s) = \frac{s^2 + s - 6}{(s^2 + 2s + 1.25)(s + 5)}$$

plot the location of the transfer function poles and zeros.



$$s^2 + s - 6 = (s+3)(s-2)$$

$$s^2 + 2s + 1.25 = 0$$

$$\frac{-2 \pm \sqrt{4 - 4(1.25)}}{2}$$

$$\frac{-2 \pm \sqrt{4 - 5}}{2}$$

$$\frac{-2 \pm j}{2} = -1 \pm j/2$$

Problem immense

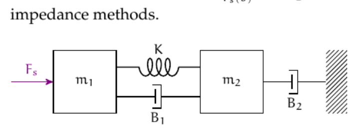
_____/35 p.

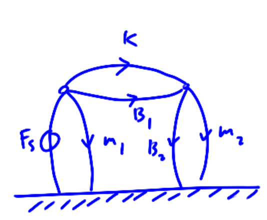
Given the system show below with a force input F_s:

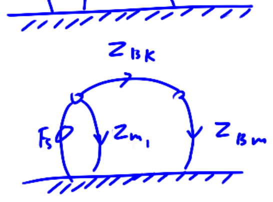
a draw a linear graph for the system,

b determine the system input impedance, and

c find the transfer function $\frac{V_{m_2}(s)}{F_s(s)}$ using impedance methods.







$$Z_{BK} = \frac{1}{\frac{1}{Z_{B_1}} + \frac{1}{Z_K}}$$

$$Z_{Bm} = \frac{1}{\frac{1}{Z_{m_2}} + \frac{1}{Z_{B_2}}}$$

$$Z_{in} = \frac{1}{\frac{1}{Z_{m_1}} + \frac{1}{Z_{BK} + Z_{Bm}}}$$

$$\frac{F_s(s)}{V_s(s)} = \frac{1}{Z_{in}}$$

$$V_{m_2} = V_s\,\frac{Z_{Bm}}{Z_{BK} + Z_{Bm}}$$

$$V_{m_2} = F_s\,\frac{1}{Z_{in}}\,\frac{Z_{Bm}}{Z_{BK} + Z_{Bm}}$$

$$\frac{V_{m_2}}{F_s} = \frac{1}{Z_{in}}\,\frac{Z_{Bm}}{Z_{BK} + Z_{Bm}}$$