Problem quantizer

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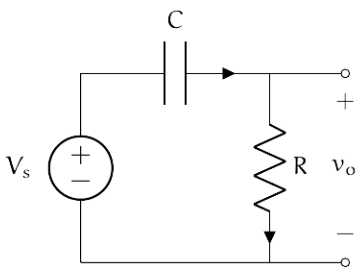
Given the circuit below, find the steady state output voltage v_o in trigonometric form, $v_o = B\cos(\omega t + \phi)$, with the input voltage, $V_s = A\cos(\omega t + \frac{\pi}{4})$. For this problem:

a find the output amplitude B, and
b determine the output phase ϕ.



$Z_R = R$

$Z_C = \frac{1}{j\omega C}$

$V_o = V_s \frac{Z_R}{Z_C + Z_R} = V_s Z$

$Z = \frac{R}{\frac{1}{j\omega C} + R} = \frac{j\omega RC}{1 + j\omega RC} \frac{(1 - j\omega RC)}{(1 - j\omega RC)}$

$= \frac{j\omega RC + \omega^2 R^2 C^2}{1 + \omega^2 R^2 C^2}$

$|Z| = \frac{\sqrt{\omega^4 R^4 C^4 + \omega^2 R^2 C^2}}{1 + \omega^2 R^2 C^2}$

$\angle(Z) = \tan^{-1}\left(\frac{\omega RC}{\omega^2 R^2 C^2}\right)$

$= \tan^{-1}\left(\frac{1}{\omega RC}\right)$

$Z = |Z| e^{\angle(Z)}$

$V_s = A\cos(\omega t + \pi/4) = Ae^{\pi/4}$

$V_o = V_s Z = Ae^{\pi/4}|Z|e^{\angle(Z)}$

$= A|Z|e^{\pi/4 + \angle(Z)}$

$= A|Z|\cos(\omega t + \pi/4 + \angle(Z))$

$B = A|Z|$

$\boxed{B = A\frac{\omega RC\sqrt{\omega^2 R^2 C^2 + 1}}{1 + \omega^2 R^2 C^2}}$

$\phi = \pi/4 + \angle(Z)$

$\boxed{\phi = \frac{\pi}{4} + \tan^{-1}\left(\frac{1}{\omega RC}\right)}$

Problem levitation

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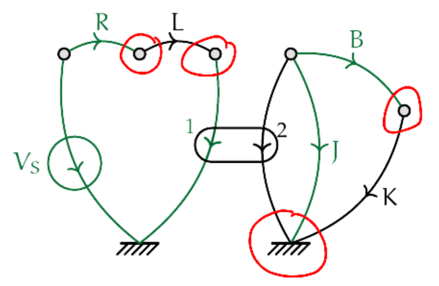
In the linear graph and normal tree below a system is depicted consisting of a motor driven by a voltage source V_S with inertia J driving a rotary damper and spring connected in series. Let the motor constant be K_a, and outputs of the system be the rotational velocity of the inertia, Ω_J, and the change in rotational velocity across the rotational damper, Ω_B.
Given this linear graph and normal tree:

a determine the state variables,
b define the state, input, and output vectors,
c write the elemental, continuity, and compatibility equations, and
d solve for the state and output equations.



State Ω_J i_L τ_K

$x = \begin{bmatrix} i_L \\ \Omega_J \\ \tau_K \end{bmatrix}$ $u = [V_s]$ $y = \begin{bmatrix} \Omega_J \\ \Omega_B \end{bmatrix}$

Primary: V_R V_1 V_s i_L Ω_J Ω_B τ_K τ_2

Elemental:

$V_R = R i_R = R i_L$

$\frac{di_L}{dt} = \frac{1}{L} V_L = \frac{1}{L}(V_s - V_R - V_1) = \frac{1}{L}(V_s - R i_L - K_a \Omega_J)$

$\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} \Omega_2 \\ \tau_2 \end{bmatrix} \begin{bmatrix} K_a & 0 \\ 0 & -1/K_a \end{bmatrix}$ $V_1 = \Omega_2 K_a = K_a \Omega_J$
$\tau_2 = -K_a i_1 = -K_a i_L$

$\frac{d\Omega_J}{dt} = \frac{1}{J}\tau_J = \frac{1}{J}(-\tau_2 - \tau_K) = \frac{1}{J}(K_a i_L - K(\Omega_J - \Omega_B))$

$\Omega_B = \frac{1}{B}\tau_B = \frac{1}{B}\tau_K$ $= \frac{1}{J}(K_a i_L - K(\Omega_J - \frac{1}{B}\tau_K))$

$\frac{d\tau_K}{dt} = K\Omega_K = K(\Omega_J - \Omega_B) = K(\Omega_J - \frac{1}{B}\tau_K)$

Continuity:
$i_R = i_L$
$i_1 = i_L$
$\tau_B = \tau_K$
$\tau_J = -\tau_2 - \tau_K$

Compatibility:
$V_L = V_s - V_R - V_1$
$\Omega_2 = \Omega_J$
$\Omega_K = \Omega_J - \Omega_B$

$\dot{x} = \begin{bmatrix} \frac{di_L}{dt} \\ \frac{d\Omega_J}{dt} \\ \frac{d\tau_K}{dt} \end{bmatrix}$

$\dot{x} = \underbrace{\begin{bmatrix} -R/L & -K_a/L & 0 \\ K_a/J & -K/J & \frac{K}{JB} \\ 0 & K & -\frac{K}{B} \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 1/L \\ 0 \\ 0 \end{bmatrix}}_{B} u$ $[V_s]$

$y = \begin{bmatrix} \Omega_J \\ \Omega_B \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1/B \end{bmatrix}}_{C} \begin{bmatrix} i_L \\ \Omega_J \\ \tau_K \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{D} [V_s]$