lti.super+ Superposition, derivative, and integral properties

1 From the principle of superposition, linear, time invariant (LTI) system responses to both initial conditions and nonzero forcing can be obtained by summing the free response y_{fr} and forced response y_{fo}:

superposition

linear, time−invariant (LTI) systems

$$y(t) = y_{fr}(t) + y_{fo}(t).$$

Moreover, superposition says that any linear combination of inputs yields a corresponding linear combination of outputs. That is, we can find the response of a system to each input, separately, then linearly combine (scale and sum) the results according to the original linear combination. That is, for inputs u_1 and u_2 and constants $a_1, a_2 \in \mathbb{R}$, a forcing function

$$\boxed{f(t) = a_1 u_1(t) + a_2 u_2(t)}$$

would yield output

$$\boxed{y(t) = a_1 y_1(t) + a_2 y_2(t)}$$

where y_1 and y_2 are the outputs for inputs u_1 and u_2, respectively.

$f(t) = 4 + 5\cos(\omega t)$

$f(t) = 3 + 5\cos(\omega t)$

$u_1(t) = 1 \longrightarrow y_1(t)$

$u_2(t) = \cos(\omega t) \longrightarrow y_2(t)$

$y(t) = 4 y_1(t) + 5 y_2(t)$

$y(t) = 3 y_1(t) + 5 y_2(t)$

2 This powerful principle allows us to construct solutions to complex forcing functions by decomposing the problem. It also allows us to make extensive use of existing solutions to common inputs.

3 There are two more LTI system properties worth noting here. Let a system have input u_1 and corresponding output y_1. If the system is then given input $u_2(t) = \dot{u}_1(t)$, the corresponding output is

$u_1(t) = \cos(t)$

$u_2(t) = \sin(t)$

$\dot{u}_1(t) = u_2(t)$

$$\boxed{y_2(t) = \dot{y}_1(t)}$$

Similarly, if the same system is then given input $u_3(t) = \int_0^t u_1(\tau) d\tau$, the corresponding output is

$$\boxed{y_3(t) = \int_0^t y_1(\tau)\, d\tau}$$

$u_1(t) = 1$

$u_3(t) = t$

These are sometimes called the derivative and integral properties of LTI systems.

derivative property

integral property

$$\int_0^t u_1(\tau)\, d\tau = \int_0^t 1\, d\tau = \tau\Big|_0^t = t$$

$u(t) = 1 + t$

$y_1(t)$

$y_3(t) = \int_0^t y_1(\tau)\, d\tau$