lti.exe Exercises for Chapter lti

Exercise lti.oil

_____/10 p.

A certain sensor used to measure displacement over time t is tested several times with input displacement $u_1(t)$ and a certain function $y_1(t)$ is estimated to properly characterize the corresponding voltage output.
Assuming the sensor is linear and time-invariant, what would we expect the output sensor voltage $y_2(t)$ to be when the following input is applied?

$$u_2(t) = 3\,\dot{u}_1(t) - 5\,u_1(t) + \int_0^t 6\,u_1(\tau)\,d\tau \tag{1}$$

Exercise lti.water

_____/15 p.

A system with input $u(t)$ and output $y(t)$ has the governing dynamical equation

$$2\,\ddot{y} + 12\,\dot{y} + 50\,y = -10\dot{u} + 4u. \tag{2}$$

a. What is the equilibrium $y(t)$ when $u(t) = 6$?
b. Demonstrate the stability, marginal stability, or instability of the system.

Qualities of transient response

1 In this chapter, we explore the qualities of transient response—the response of the system in the interval during which initial conditions dominate.
2 We focus on characterizing first- and second-order linear systems; not because they're easiest (they are), but because nonlinear systems can be linearized about an operating point and because higher-order linear system responses are just sums of first- and second-order responses, making "everything look first- and second-order." Well, many things, at least.

linearization
operating point

$\dot{x} = Ax + Bu$
$y = Cx + Du$

3 In this chapter, we primarily consider systems represented by single-input, single-output (SISO) ordinary differential equations (also called io ODEs)—with variable y representing the output, dependent variable time t, variable u representing the input, forcing function f, constant coefficients a_i, b_j, order n, and $m \leqslant n$ for $n \in \mathbb{N}_0$—of the form

SISO

forcing function

$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \cdots + a_1\frac{dy}{dt} + a_0 y = f, \text{ where} \tag{1a}$$

$$f \equiv b_m\frac{d^m u}{dt^m} + b_{m-1}\frac{d^{m-1}u}{dt^{m-1}} + \cdots + b_1\frac{du}{dt} + b_0 u. \tag{1b}$$

Note that the forcing function f is related to but distinct from the input u. This terminology proves rather important.