State–space response

1 Recall that, for a state-space model, the state $\mathbf{x}$, input $\mathbf{u}$, and output $\mathbf{y}$ vectors interact through two equations:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) \quad (1a)$$
$$\mathbf{y} = \mathbf{g}(\mathbf{x}, \mathbf{u}, t) \quad (1b)$$

where $\mathbf{f}$ and $\mathbf{g}$ are vector-valued functions that depend on the system. Together, they comprise what is called a state-space model of a system.

state–space model

2 In accordance with the definition of a state-determined system, given an initial condition $\mathbf{x}(t_0)$ and input $\mathbf{u}$, the state $\mathbf{x}$ is determined for all $t \geqslant t_0$. Determining the state response requires the solution—analytic or numerical—of the vector differential equation Eq. 1a.

3 The second equation (1b) is algebraic. It expresses how the output $\mathbf{y}$ can be constructed from the state $\mathbf{x}$ and input $\mathbf{u}$. This means we must first solve the state equation (1a) for $\mathbf{x}$, then the output $\mathbf{y}$ is given by Eq. 1b.

4 Just because we know that, for a state-determined system, there exists a solution to Eq. 1a, doesn't mean we know how to find it. In general, $\mathbf{f} : \mathbb{R}^n \times \mathbb{R}^r \times \mathbb{R} \to \mathbb{R}^n$ and $\mathbf{g} : \mathbb{R}^n \times \mathbb{R}^r \times \mathbb{R} \to \mathbb{R}^m$ can be nonlinear functions.[1] We don't know how to solve most nonlinear state equations analytically. An additional complication can arise when, in addition to states and inputs, system parameters are themselves time-varying (note the explicit time t argument of $\mathbf{f}$ and $\mathbf{g}$). Fortunately, often a linear, time-invariant (LTI) model is sufficient.

1. Technically, since $\mathbf{x}$ and $\mathbf{u}$ are themselves functions, $\mathbf{f}$ and $\mathbf{g}$ are functionals.

5 Recall that an LTI state-space model is of the form

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x} + B\mathbf{u} \quad (2a)$$
$$\mathbf{y} = C\mathbf{x} + D\mathbf{u}, \quad (2b)$$

where A, B, C, and D are constant matrices containing system lumped-parameters such as mass or inductance. See Chapter ss for details on the derivation of such models.

6 In this chapter, we learn to solve Eq. 2a for the state response and substitute the result into Eq. 2b for the output response. First, we learn an analytic solution technique. Afterward, we learn simple software tools for numerical solution techniques.