10.20. For the rotational system described in Example 9.8, with parameter values J=1 kg-m², K=9 N-m/rad, and B=0.6 N-s/rad

- (a) Compute the state transition matrix for the system, based on a time increment of 0.2 s.
- (b) Compute and plot the solution for (i) the flywheel angular velocity Ω_J and (ii) the spring torque T_K as a function of time when the motor speed increases suddenly from 0 to 50 rad/s and then remains constant.
- (c) Compute and plot the response of
 - i. the flywheel speed
 - ii. the spring torque

as a function of time when the motor speed increases from 0 to 50 rad/s in a linear ramp over a period of 10 s and then remains constant.

- (d) Compare the two solutions with respect to
 - i. the maximum torque in the spring
 - ii. the maximum flywheel angular velocity
 - iii. the steady-state angular velocity and torque

Example 9.8

A rotational system consists of an inertial load J mounted in viscous bearings B and driven by an angular velocity source $\Omega_{in}(t)$ through a long light shaft with significant torsional stiffness K, as shown in Fig. 9.16. Derive a pair of second-order differential equations for the variables Ω_J and Ω_K .

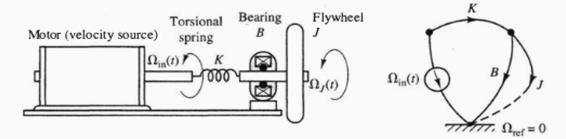


Figure 9.16: Rotational system for Example 9.8.

Solution The state variables are Ω_J and T_K , and the state equations are

$$\begin{bmatrix} \dot{\Omega}_j \\ \dot{T}_k \end{bmatrix} = \begin{bmatrix} -B/J & 1/J \\ -K & 0 \end{bmatrix} \begin{bmatrix} \Omega_J \\ T_K \end{bmatrix} + \begin{bmatrix} 0 \\ K \end{bmatrix} \Omega_{\text{in}}$$
 (i)