```
J = 1;
K = 9;
b = 0.6;
```

```
A = [-b / J, 1 / J; -K, 0]
```

```
A = 2×2
   -0.6000    1.0000
   -9.0000         0
```

```
B = [0; K]
```

```
B = 2×1
     0
     9
```

```
[M, D] = eig(A)
```

```
M = 2×2 complex
   0.0316 - 0.3146i   0.0316 + 0.3146i
   0.9487 + 0.0000i   0.9487 + 0.0000i
D = 2×2 complex
  -0.3000 + 2.9850i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.3000 - 2.9850i
```

```
dt = 0.2;
Phi_p = diag(exp(diag(D * dt)))
```

```
Phi_p = 2×2 complex
   0.7789 + 0.5294i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.7789 - 0.5294i
```

```
Phi = real(M * Phi_p * inv(M))
```

```
Phi = 2×2
    0.7257    0.1774
   -1.5963    0.8321
```

```
real(M * diag(exp(diag(D * 2 * dt))) * inv(M))
```

```
ans = 2×2
    0.2435    0.2763
   -2.4866    0.4092
```

```
Phi^2
```

```
ans = 2×2
    0.2435    0.2763
   -2.4866    0.4092
```

```
syms t T
u = 50;
M_inv = inv(M);
```

```
Phi = @(t) M * diag(exp(diag(D * t))) * M_inv;
x_fo = @(t) int(Phi(t - T) * B * u, T, 0, t);
x_fo(t)
```

```
ans =
```

$$\left(\begin{array}{c} 50 - \frac{5\,\sqrt{2}\,\sqrt{5}\,\sqrt{10}\,\sigma_1\,e^{\sigma_2}}{2} - \frac{5\,\sqrt{2}\,\sqrt{5}\,\sqrt{10}\,\sigma_1\,e^{-\sigma_2}}{2} - \frac{5\,\sqrt{5}\,\sqrt{10}\,\sqrt{22}\,\sigma_1\,e^{-\sigma_2}\,i}{66} + \frac{5\,\sqrt{5}\,\sqrt{10}\,\sqrt{22}\,\cdots}{66} \\ 30 - 15\,\sigma_3 - 15\,\sigma_4 - \frac{245\,\sqrt{2}\,\sqrt{22}\,\sigma_4\,i}{22} + \frac{245\,\sqrt{2}\,\sqrt{22}\,\sigma_3\,i}{22} \end{array} \right.$$

where

$$\sigma_1 = e^{-\frac{3\,t}{10}}$$

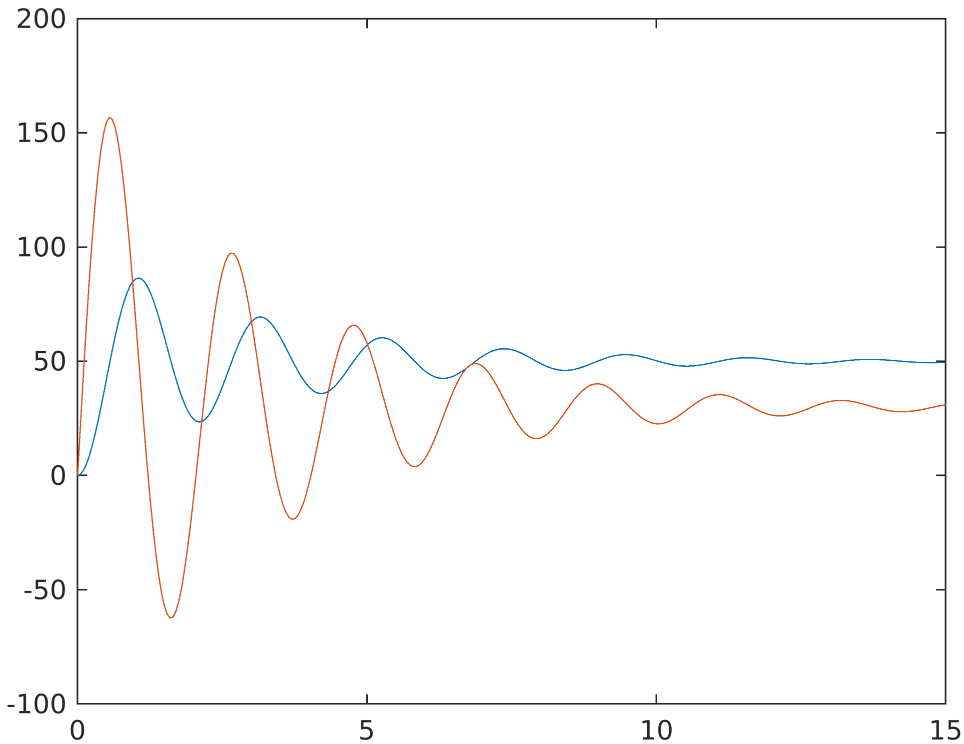
$$\sigma_2 = \frac{9\,\sqrt{11}\,t\,i}{10}$$

$$\sigma_3 = e^{-t\,\left(\frac{3}{10}+\sigma_5\right)}$$

$$\sigma_4 = e^{t\,\left(-\frac{3}{10}+\sigma_5\right)}$$

$$\sigma_5 = \frac{9\,\sqrt{11}\,i}{10}$$

```
t_vec = 0:0.05:15;
x_fun = matlabFunction(x_fo(t));
plot(t_vec, x_fun(t_vec))
```




```
syms tu
u = @(t) subs(piecewise(tu<10, 5 * tu, tu>=10, 50), tu, t);
x_fo = @(t) int(Phi(t - T) * B * u(T), T, 0, t);
x_fo(t)
```

```
ans =
```

$$\left(\begin{cases} 50 - 25\,\sigma_{12}\,\sigma_1\,e^3\,e^{-\frac{\sigma_{10}}{10}} + \frac{\sqrt{2}\ \sqrt{5}\ \sqrt{10}\ \sigma_1\,e^{-\frac{\sigma_{10}}{10}}}{120} + \frac{\sqrt{2}\ \sqrt{5}\ \sqrt{10}\ \sigma_1\,e^{\frac{\sigma_{10}}{10}}}{120} + \frac{\sqrt{5}\ \sqrt{10}\ \sqrt{11}\ \sqrt{22}\ \sigma_1\,e^{-\frac{\sigma_{10}}{10}}}{1320} \end{cases}\right.$$

where

$$\sigma_1 = e^{-\frac{3t}{10}}$$

$$\sigma_2 = e^{-9\sqrt{11}\,i}$$

$$\sigma_3 = e^{18\sqrt{11}\,i}$$

$$\sigma_4 = e^{\frac{3t}{10}}$$

$$\sigma_5 = e^{-\frac{3t}{10}-\frac{\sigma_{10}}{10}}$$

$$\sigma_6 = e^{-\frac{3t}{10}+\frac{\sigma_{10}}{10}}$$

$$\sigma_7 = e^{-\left(\frac{3}{10}+\sigma_{11}\right)\,(t-10)}$$

$$\sigma_8 = e^{\left(-\frac{3}{10}+\sigma_{11}\right)\,(t-10)}$$

$$\sigma_9 = 148\ \sqrt{22}\ \sigma_{12}\,i$$

$$\sigma_{10} = 9\ \sqrt{11}\ t\,i$$

$$\sigma_{11} = \frac{9\ \sqrt{11}\ i}{10}$$

```
x_fun = @(t) subs(x_fo(tu), tu, t)
```

```
x_fun = function_handle with value:
    @(t)subs(x_fo(tu),tu,t)
```

```
plot(t_vec, x_fun(t_vec))
```

```
Warning: Imaginary parts of complex X and/or Y arguments ignored.
```

```
hold on
plot(t_vec, u(t_vec))
```