RW 10.5

$$\Phi(t) = \begin{bmatrix} e^{-2t} & 0 \\ e^{-t} - e^{-2t} & e^{-t} \end{bmatrix} \qquad X(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

a) find $\Phi(0.2)$

$$\Phi(0.2) = \begin{bmatrix} e^{-2(0.2)} & 0 \\ e^{-0.2} - e^{-2(0.2)} & e^{-0.2} \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 0.67 & 0 \\ 0.15 & 0.81 \end{bmatrix}}$$

b) find $X(t)$ for t from 0 to 1 using part a in steps of 0.2

$$X(0.2) = \begin{bmatrix} 0.67 & 0 \\ 0.15 & 0.81 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.67 \\ 0.96 \end{bmatrix}$$

$$X(0.4) = (\Phi(0.2))^2 X(0)$$

$$= \Phi(0.2)\, \underbrace{\Phi(0.2)\, X(0)}_{X(0.2)}$$

$$= \begin{bmatrix} 0.67 & 0 \\ 0.15 & 0.81 \end{bmatrix} \begin{bmatrix} 0.67 \\ 0.96 \end{bmatrix} = \begin{bmatrix} 0.45 \\ 0.88 \end{bmatrix}$$

$$X(0.6) = (\Phi(0.2))^3 X(0)$$

$$= \Phi(0.2)\, \underbrace{(\Phi(0.2))^2 X(0)}_{X(0.4)}$$

$$= \begin{bmatrix} 0.67 & 0 \\ 0.15 & 0.81 \end{bmatrix} \begin{bmatrix} 0.45 \\ 0.88 \end{bmatrix} = \begin{bmatrix} 0.3 \\ 0.78 \end{bmatrix}$$