Find $\frac{d}{dt}\cos(\omega_0 t)$ using Fourier Transform

$$y(t) = \cos(\omega_0 t)$$

$$Y(\omega) = \int_{-\infty}^{\infty} \cos(\omega_0 t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) e^{-j\omega t}\, dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} e^{j(\omega_0 - \omega)t} + e^{j(-\omega_0 - \omega)t}\, dt$$

$$= \frac{1}{2}\left(\frac{e^{j(\omega_0 - \omega)t}}{j(\omega_0 - \omega)} + \frac{e^{j(-\omega_0 - \omega)t}}{j(-\omega_0 - \omega)}\right)\Bigg|_{-\infty}^{\infty}$$

$$Y(\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$$

$$y'(t) = F^{-1}(j\omega Y(\omega)) = F^{-1}(j\omega\pi(\delta(\omega - \omega_0) + \delta(\omega + \omega_0)))$$

$$F^{-1}(-\omega(-j\pi\delta(\omega - \omega_0) - j\pi\delta(\omega + \omega_0)))$$

$$= -\omega F^{-1}(-j\pi\delta(\omega - \omega_0) - j\pi\delta(\omega + \omega_0))$$

$$y'(t) = -\omega\sin(\omega_0 t)$$