

RW 10.12  
part c.

$$A = \begin{bmatrix} 0 & 1 \\ -2s & 0 \end{bmatrix} \quad \text{find } \Phi(t)$$

find  $\lambda$ :

$$\det(\lambda I - A) = 0$$

$$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2s & 0 \end{bmatrix}\right) = 0$$

$$\begin{vmatrix} \lambda & -1 \\ 2s & \lambda \end{vmatrix} = 0$$

$$\lambda^2 + 2s = 0$$

$$\lambda^2 = -2s$$

$$\lambda = \pm 5i$$

$$A = \begin{bmatrix} 5i & 0 \\ 0 & -5i \end{bmatrix}$$

find  $m$ :

$$(\lambda_i I - A)m_i = 0$$

$$\left(\begin{bmatrix} 5i & 0 \\ 0 & 5i \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2s & 0 \end{bmatrix}\right) m_1 = \vec{0}$$

$$\begin{bmatrix} 5i & -1 \\ 2s & 5i \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$5i m_{11} - m_{12} = 0 \quad 5i m_{11} = m_{12}$$

$$2s m_{11} + 5i m_{12} = 0 \quad m_1 = \begin{bmatrix} 1 \\ 5i \end{bmatrix}$$

$$2s + 5i(5i) = 0$$

$$2s - 2s = 0 \quad \checkmark$$

$$\begin{bmatrix} -5i & -1 \\ 2s & -5i \end{bmatrix} \begin{bmatrix} m_{21} \\ m_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-5i m_{21} - m_{22} = 0 \quad -5i m_{21} = m_{22}$$

$$2s m_{21} - 5i m_{22} = 0 \quad m_2 = \begin{bmatrix} 1 \\ -5i \end{bmatrix}$$

$$2s - 5i(-5i) = 0$$

$$2s - 2s = 0 \quad \checkmark$$

$$M = \begin{bmatrix} 1 & 1 \\ 5i & -5i \end{bmatrix}$$

find  $\Phi'(t)$

$$\Phi'(t) = \begin{bmatrix} e^{5it} & 0 \\ 0 & e^{-5it} \end{bmatrix}$$

find  $\Phi(t)$

$$\Phi(t) = M \Phi'(t) M^{-1}$$

$$\det(M) = \begin{vmatrix} 1 & 1 \\ 5i & -5i \end{vmatrix}$$

$$= -5i - 5i = -10i$$

$$\begin{bmatrix} 1 & 1 \\ 5i & -5i \end{bmatrix} \begin{bmatrix} 5i & -1 \\ -5i & 1 \end{bmatrix} = \begin{bmatrix} -10i & 0 \\ 0 & -10i \end{bmatrix}$$

$$M^{-1} = \frac{1}{\det(M)} \begin{bmatrix} -5i & -1 \\ -5i & 1 \end{bmatrix}$$

$$= \frac{1}{-10i} \begin{bmatrix} -5i & -1 \\ -5i & 1 \end{bmatrix}$$

$$\Phi(t) = \begin{bmatrix} 1 & 1 \\ 5i & -5i \end{bmatrix} \begin{bmatrix} e^{5it} & 0 \\ 0 & e^{-5it} \end{bmatrix} \frac{1}{-10i} \begin{bmatrix} -5i & -1 \\ -5i & 1 \end{bmatrix}$$

$$= \frac{1}{-10i} \begin{bmatrix} e^{5it} & e^{-5it} \\ 5ie^{5it} & -5ie^{-5it} \end{bmatrix} \begin{bmatrix} -5i & -1 \\ -5i & 1 \end{bmatrix}$$

$$= \frac{1}{-10i} \begin{bmatrix} -5ie^{5it} & -5ie^{-5it} & -e^{5it} & -e^{-5it} \\ 25e^{5it} & -25e^{-5it} & -5ie^{5it} & 5ie^{-5it} \end{bmatrix}$$

$$x_{fr}(t) = \Phi(t) x(0)$$

$$y_{fr}(t) = B x_{fr}(t) = B \Phi(t) x(0)$$