RW 8.21 $\quad \frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 3u_s(t) \qquad y(0) = 1$

use table seconds. 1

$$\frac{d^2y}{dt^2} + 2\zeta\omega_n \frac{dy}{dt} + \omega_n^2 y = 3u_s(t)$$

find ω_n $\quad \omega_n^2 = 5 \quad \underline{\omega_n = \sqrt{5}} \qquad \sqrt{5} > 1$

find ζ $\quad 2\zeta\omega_n = 2$

$$\zeta = \frac{2}{2\omega_n} = \frac{1}{\omega_n} = \frac{1}{\sqrt{5}} < 1$$

find $y_{fr}(t)$

$$y_{fr}(t) = y(0)\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t + \psi)$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = \sqrt{5}\sqrt{1-\tfrac{1}{5}} = \sqrt{5-1} = \sqrt{4} = 2$$

$$\psi = -\operatorname{atan}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right) = -\operatorname{atan}\left(\frac{1/\sqrt{5}}{\sqrt{1-1/5}}\right)$$

$$= -\operatorname{atan}\left(\frac{1/\sqrt{5}}{\sqrt{4/5}}\right) = -\operatorname{atan}\left(\frac{1}{\sqrt{4}}\right)$$

$$= -\operatorname{atan}\left(\frac{1}{2}\right) = -0.46$$

$$y_{fr}(t) = \frac{e^{-1/\sqrt{5}\sqrt{5}\,t}}{\sqrt{4/5}}\cos(2t - 0.46)$$

$$= \frac{\sqrt{5}e^{-t}}{2}\cos(2t - 0.46)$$

find $y_{fo}(t)$

$$y_c(t) = \frac{1}{\omega_n^2}\left(1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t + \psi)\right)$$

$$= \frac{1}{5}\left(1 - \frac{\sqrt{5}e^{-t}}{2}\cos(2t - 0.46)\right)$$

$$y_{fo}(t) = 3\,y_c(t)$$

find $y(t)$

$$y(t) = y_{fr}(t) + y_{fo}(t)$$

$$= \frac{\sqrt{5}e^{-t}}{2}\cos(2t-0.46) + \frac{3}{5}\left(1 - \frac{\sqrt{5}e^{-t}}{2}\cos(2t-0.46)\right)$$

$$= \left(1 - \frac{3}{5}\right)\frac{\sqrt{5}e^{-t}}{2}\cos(2t-0.46) + \frac{3}{5}$$

$$= \frac{2}{5}\,\frac{\sqrt{5}e^{-t}}{2}\cos(2t-0.46) + \frac{3}{5}$$

$$= \boxed{\frac{e^{-t}}{\sqrt{5}}\cos(2t-0.46) + \frac{3}{5}}$$