

**ME 370 - System Dynamics and Control**

Midterm Exam 1

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Contents

canada . . . . .	1
argentina . . . . .	1
morocco . . . . .	2
taiwan . . . . .	2

Directions: take-home, open notes, open book. Use your own paper, work neatly, and clearly mark your answers. MATLAB and other programming languages may not be used. Partial credit may be given. Submit as a single pdf.

For 5 points of extra credit, participate in this study on why students choose to study STEM related subjects before data collection closes on February 18th.

<https://tinyurl.com/ThesisSTEMSurvey>

**Problem canada**

Given a differential equation, \_\_\_\_\_/30 p.

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 25y = f(t),$$

with initial conditions  $\frac{dy}{dt}|_{t=0} = 0$  and  $y(0) = 8$ , find

- a the undamped natural frequency  $\omega_n$  and damping ratio  $\zeta$ ,
- b the free response  $y_f(t)$ ,
- c the forced response due to a Dirac delta forcing function  $f(t) = \delta(t)$ ,
- d the forced response due to a unit step forcing function  $f(t) = u_s(t)$ ,
- e the forced response due to a unit ramp forcing function  $f(t) = r(t)$ ,
- f the forced response to the forcing function,

$$f(t) = 7\delta(t) - 4u_s(t) + 6r(t),$$

and  
g the total response from the initial condition and the forcing function in part f.

$$\frac{d^2y}{dt^2} + 2\zeta\omega_n\frac{dy}{dt} + \omega_n^2y = f(t)$$

$$\omega_n^2 = 25 \Rightarrow \omega_n = 5 \quad \zeta = 2\zeta\omega_n = 2\zeta 5$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 5\sqrt{1-0.3^2} = 4.77$$

$$\frac{3}{2.5} = \zeta = 0.3$$

$$\psi = -\tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right) = -\tan^{-1}\left(\frac{0.3}{\sqrt{1-0.3^2}}\right) = -0.3$$

$$y_{fr}(t) = y(0)\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \cos(\omega_d t + \psi) = 8\frac{e^{-0.3 \cdot 5 t}}{\sqrt{1-0.3^2}} \cos(4.77t - 0.3)$$

$$= 8.39 e^{-1.5t} \cos(4.77t - 0.3)$$

$$y_{\delta}(t) = \frac{e^{-\zeta\omega_n t}}{\omega_n\sqrt{1-\zeta^2}} \sin(\omega_d t) = \frac{e^{-0.3 \cdot 5 t}}{5\sqrt{1-0.3^2}} \sin(4.77t)$$

$$= 0.21 e^{-1.5t} \sin(4.77t)$$

$$y_s(t) = \frac{1}{\omega_n^2} \left(1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \cos(\omega_d t + \psi)\right) = 0.04 - 0.042 e^{-1.5t} \cos(4.77t - 0.3)$$

$$y_r(t) = \frac{1}{\omega_n^2} \left(t + \frac{e^{-\zeta\omega_n t}}{\omega_n} \left(2\zeta \cos(\omega_d t) + \frac{2\zeta^2 - 1}{\sqrt{1-\zeta^2}} \sin(\omega_d t)\right) - \frac{2\zeta}{\omega_n}\right)$$

$$= 0.04t + e^{-1.5t} (0.0048 \cos(4.77t) - 0.0061 \sin(4.77t)) - 0.0048$$

$$y_{f0}(t) = 7y_{\delta}(t) - 4y_s(t) + 6y_r(t)$$

$$= 7(0.21 e^{-1.5t} \sin(4.77t))$$

$$- 4(0.04 - 0.042 e^{-1.5t} \cos(4.77t - 0.3))$$

$$+ 6(0.04t + e^{-1.5t} (0.0048 \cos(4.77t) - 0.0061 \sin(4.77t)) - 0.0048)$$

$$= 0.024t - 1.17 + e^{-1.5t} (1.43 \sin(4.77t) + 0.029 \cos(4.77t) - 0.163 \cos(4.77t - 0.3))$$

$$y(t) = y_{f0}(t) + y_{fr}(t)$$

$$= 0.024t - 1.17 + e^{-1.5t} (1.43 \sin(4.77t) + 0.029 \cos(4.77t) + 8.22 \cos(4.77t - 0.3))$$

$$= 0.024t - 1.17 + 7.15 e^{-1.5t} \sin(4.77t + 1.7)$$

Exam p.2

**Problem argentina**

Given a state space system, \_\_\_\_\_/30 p.

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du,$$

with,

$$A = \begin{bmatrix} -8 & -6 \\ 3 & 1 \end{bmatrix} \text{ and}$$

$$C = \begin{bmatrix} 2 & -1 \end{bmatrix},$$

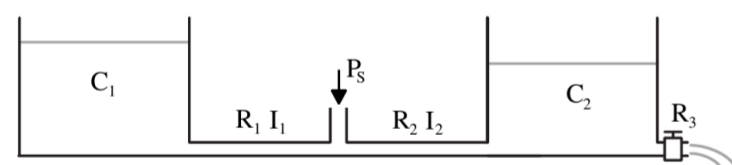
find

- a the system's Eigen values  $\lambda_i$ ,
- b the Eigen vectors  $m_i$  and modal matrix  $M$ ,
- c the diagonalized state transition matrix  $\Phi'(t)$ ,
- d the state transition matrix in the original basis  $\Phi(t)$ , and
- e the output free response  $y_f(t)$  due to an initial condition  $x(0) = [4, -1]^T$ .

**Problem morocco**

For the system below with a pressure source  $P_s$ , fluid resistances  $R_i$ , fluid inertances  $I_i$ , and fluid capacitances  $C_i$ , find

- a the linear graph,
- b the normal tree, and
- c the system state variables and system order.



\_\_\_\_\_/20 p.

Exam p.3

**Problem taiwan**

The cooling system for a desktop computer CPU is shown below. We can consider the CPU as a heat flow source  $Q_c$ . A thermal interface material is then used to transfer heat from the CPU to the cooling system. However, this thermal interface material is not a perfect thermal conductor. The cooling system then consists of a base plate with thermal capacitance and a "heat pipe" which moves the heat away from the CPU. The "heat pipe" is again an imperfect thermal conductor. At the other end of the "heat pipe" is a constant temperature (which we can model as a temperature source). For this system, find

- a the linear graph,
- b the normal tree, and
- c the system state variables and system order.

