RW 14.1

$$H(s) = \frac{V(s)}{U(s)} = \frac{K}{\tau s + 1} \qquad K = 2 \qquad \tau = 0.1$$

$$u(t) = 5\sin(\omega t)$$

a) find $H(j\omega)$ and find $|H(j\omega)|$, $\angle H(j\omega)$

$$H(j\omega) = H(s)\Big|_{s \to j\omega}$$

$$= \frac{K}{\tau j\omega + 1} = \boxed{\frac{2}{0.1 j\omega + 1}} \quad \frac{1 - 0.1 j\omega}{1 - 0.1 j\omega}$$

$$= \frac{2 - 0.2 j\omega}{1 + 0.01\omega^2}$$

$$|H(j\omega)| = \sqrt{Re(H(j\omega))^2 + Im(H(j\omega))^2} = \boxed{\frac{\sqrt{4 + 0.04\omega^2}}{1 + 0.01\omega^2}}$$

$$\angle H(j\omega) = \tan^{-1}\left(\frac{Im(H(j\omega))}{Re(H(j\omega))}\right) = \tan^{-1}\left(\frac{-0.2\omega}{2}\right)$$

$$= \boxed{\tan^{-1}(-0.1\omega)}$$

b) find $y(t)$ for

$\omega = 5$ $\quad |H(j5)| = \dfrac{\sqrt{4 + 0.04(5)^2}}{1 + 0.01(5)^2} = 1.78$

$\angle H(j5) = \tan^{-1}(-0.1(5)) = -0.46$

$$y(t) = 5(1.78)\sin(5t - 0.46)$$

$\omega = 10$ $\quad |H(j10)| = 1.41$ $\quad \angle H(j10) = -0.79$

$$y(t) = 5(1.41)\sin(10t - 0.79)$$

$\omega = 40$ $\quad |H(j40)| = 0.52$ $\quad \angle H(j40) = -1.33$

$$y(t) = 5(0.52)\sin(40t - 1.33)$$