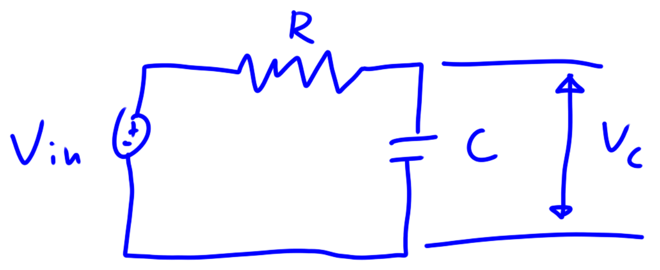
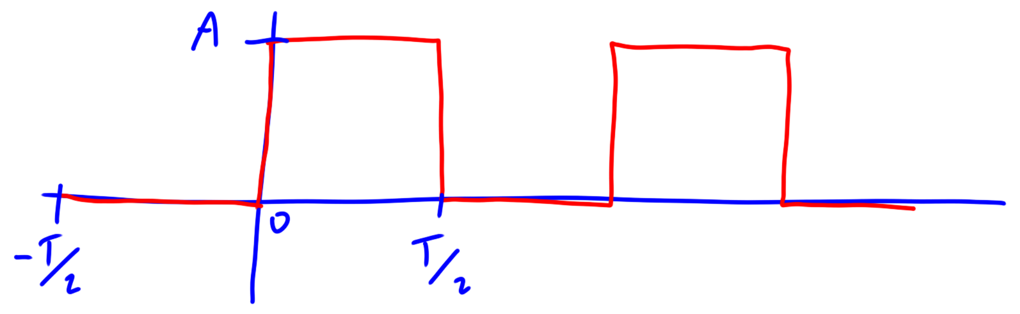
Given an RC circuit with a periodic input, find the voltage accross the capacitor



$$H(s) = \frac{1}{1+RCs} = \frac{V_c(s)}{V_{in}(s)}$$

V_{in}



$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} V_{in}(t)\cos(w_n t)\,dt = \frac{2}{T}\int_0^{T/2} A\cos(w_n t)\,dt$$

$$= \frac{2A}{T w_n}\sin(w_n t)\Big|_0^{T/2} = \frac{2A}{T w_n}\sin\left(w_n \frac{T}{2}\right)$$

$$w_n = \frac{2\pi n}{T} \qquad = \frac{2A}{T w_n}\sin\left(\frac{\cancel{2}\pi n}{\cancel{T}}\frac{\cancel{T}}{\cancel{2}}\right) = 0$$

$$a_0 = \frac{2}{T}\int_{-T/2}^{T/2} V_{in}(t)\,dt = \frac{2}{T}\int_0^{T/2} A\,dt = \frac{2A}{T}\,t\Big|_0^{T/2} = \frac{\cancel{2}A}{\cancel{T}}\frac{\cancel{T}}{\cancel{2}}$$

$$= A$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} V_{in}(t)\sin(w_n t)\,dt = \frac{2}{T}\int_0^{T/2} A\sin(w_n t)\,dt$$

$$= \frac{-2A}{T w_n}\cos(w_n t)\Big|_0^{T/2} = \frac{-2A}{T w_n}\cos\left(w_n \frac{T}{2}\right) + \frac{2A}{T w_n}$$

$$= \frac{2A}{T w_n}\left(1-\cos\left(w_n \frac{T}{2}\right)\right) = \frac{\cancel{2}A}{\cancel{T}\frac{\cancel{2}\pi n}{\cancel{T}}}\left(1-\cos\left(\frac{\cancel{2}\pi n}{\cancel{T}}\frac{\cancel{T}}{\cancel{2}}\right)\right)$$

$$= \frac{A}{n\pi}\left(1-\cos(n\pi)\right)$$

$$V_{in}(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} b_n \sin(w_n t)$$

$$H(jw) = H(s)\Big|_{s\to jw} = \frac{1}{1+RCs}\Big|_{s\to jw} = \frac{1}{1+RCjw}\,\frac{1-RCjw}{1-RCjw}$$

$$= \frac{1-RCjw}{1+(RCw)^2}$$

$$Re(H(jw)) = \frac{1}{1+(RCw)^2} \qquad Im(H(jw)) = \frac{-RCw}{1+(RCw)^2}$$

$$|H(jw)| = \sqrt{Re(H(jw))^2 + Im(H(jw))^2}$$

$$= \sqrt{\left(\frac{1}{1+(RCw)^2}\right)^2 + \left(\frac{-RCw}{1+(RCw)^2}\right)^2}$$

$$= \frac{\sqrt{1+(RCw)^2}}{1+(RCw)^2} = \frac{1}{\sqrt{1+(RCw)^2}}$$

$$\angle H(jw) = \tan^{-1}\left(\frac{Im(H(jw))}{Re(H(jw))}\right) = \tan^{-1}\left(\frac{\frac{-RCw}{\cancel{1+(RCw)^2}}}{\frac{1}{\cancel{1+(RCw)^2}}}\right)$$

$$= \tan^{-1}(-RCw)$$

$$V_C(t) = H(j0)\frac{a_0}{2} + \sum_{n=1}^{\infty} b_n |H(jw_n)| \sin(w_n t + \angle H(jw_n))$$