

Transfer functions

- 1 In Lec. ss.ss2tf2io, we briefly introduced the transfer function, a very important dynamic system representation to which we now turn our attention. We repeat the definition here.
- 2 Let a system have an input u and an output y . Let the Laplace transform of each be denoted U and Y , both functions of complex Laplace transform variable s . A transfer function H is defined as the ratio of the Laplace transform of the output over the input:

transfer function definition

$$H(s) = \frac{Y(s)}{U(s)}. \quad (1)$$

- 3 The transfer function is exceedingly useful in many types of analysis. One of its most powerful aspects is that it gives us access to thinking about systems as operating on an input u and yielding an output y .
- 4 As we learned in Lec. ss.ss2tf2io, one can easily convert a state-space model to a (matrix) transfer function model with the formula

$$H(s) = C(sI - A)^{-1}B + D. \quad (2)$$

We also learned that a transfer function and an io ODE are related via the Laplace transform. The similarities are rather easy to spot, so io ODEs and transfer functions can be converted to each other via inspection.

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$