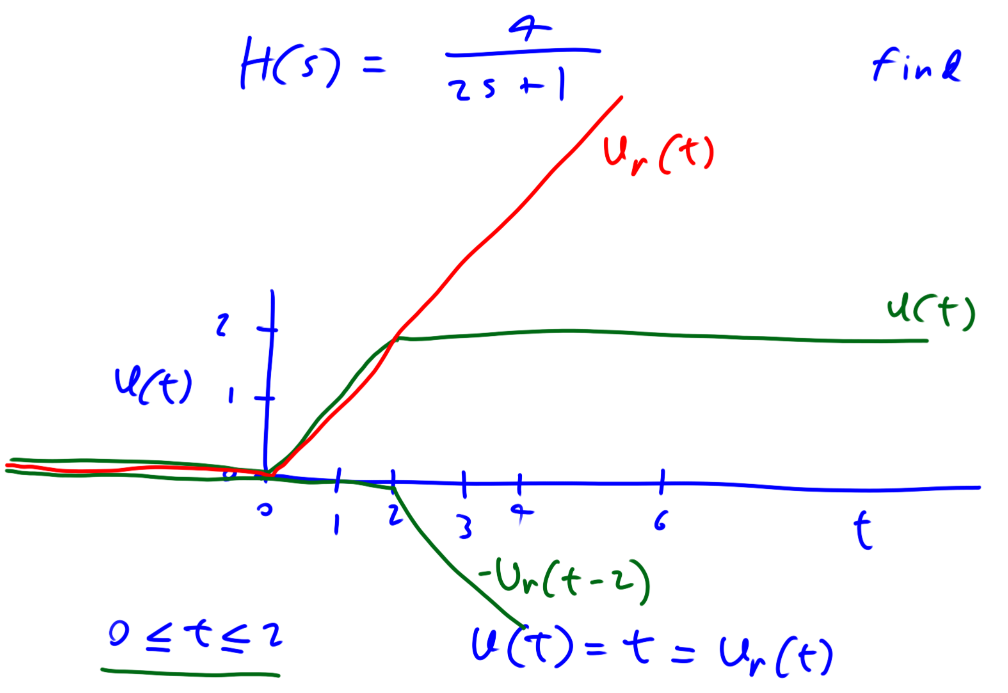
RW 15.26

$$H(s) = \frac{4}{2s+1} \qquad \text{find } y(t)$$



$U_r(1) - U_r(1-2) = 1$

$U_r(2) - U_r(2-2) = 2$

$U_r(3) - U_r(3-2) = 2$

$U_r(4) - U_r(4-2) = 2$

$\underline{0 \le t \le 2}$ $\qquad u(t) = t = u_r(t)$

$$U(s) = \mathcal{L}(u(t)) = \frac{1}{s^2}$$

$$H(s) = \frac{Y(s)}{U(s)}$$

$$Y(s) = H(s)\,U(s) = \frac{4}{2s+1}\,\frac{1}{s^2}$$

$$= \frac{16}{2s+1} - \frac{8}{s} + \frac{4}{s^2}$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{16}{2s+1} - \frac{8}{s} + \frac{4}{s^2}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{16}{2s+1}\right) - \mathcal{L}^{-1}\left(\frac{8}{s}\right) + \mathcal{L}^{-1}\left(\frac{4}{s^2}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{8}{s+\frac{1}{2}}\right) - \mathcal{L}\left(\frac{8}{s}\right) + \mathcal{L}^{-1}\left(\frac{4}{s^2}\right)$$

$$= 8e^{-t/2} - 8u_s(t) + 4u_r(t)$$

$$y(t) = 8e^{-t/2} - 8 + 4t$$

$$y(2) = 8e^{-2/2} \cancel{- 8 + 4(2)} = 8e^{-1} = 2.94$$

$t \geq 2$ $\qquad u(t) = 2 = 2u_s(t)$ $\qquad U(s) = \mathcal{L}(u(t)) = \frac{2}{s}$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{4}{2s+1}$$

$$(2s+1)Y(s) = 4U(s)$$

$$\mathcal{L}\left(2\frac{dy}{dt} + y = 4u(t)\right)$$

$$2(sY(s) - y(0)) + Y(s) = 4U(s)$$

$$2(sY(s) - 2.94) + Y(s) = 4U(s)$$

$$2sY(s) - 5.88 + Y(s) = 4U(s)$$

$$2sY(s) + Y(s) = 4U(s) + 5.88$$

$$(2s+1)Y(s) = 4U(s) + 5.88$$

$$Y(s) = \frac{4U(s) + 5.88}{2s+1} = \frac{4U(s)}{2s+1} + \frac{5.88}{2s+1}$$

$$= \frac{4}{2s+1}\,\frac{2}{s} + \frac{5.88}{2s+1}$$

$$= \frac{8}{s} - \frac{16}{2s+1} + \frac{5.88}{2s+1}$$

$$= \frac{8}{s} - \frac{10.12}{2s+1} = \frac{8}{s} - \frac{5.06}{s+\frac{1}{2}}$$

$$y(t) = 8u_s(t) - 5.06e^{-t/2}$$

time shift

$$y(t) = 8 - 5.06e^{-\frac{(t-2)}{2}}$$

Method 2

$$\mathcal{L}(u(t)) = \int_0^\infty u(t)e^{-st}\,dt$$

$$= \int_0^2 te^{-st}\,dt + \int_2^\infty 2e^{-st}\,dt$$

$$= \left.\frac{-e^{-st}(st+1)}{s^2}\right|_0^2 + \left.\frac{-2e^{-st}}{s}\right|_2^\infty$$

$$= \frac{-e^{-2s}(2s+1)}{s^2} - \frac{-1}{s^2} + \cancel{\lim_{t\to\infty}\frac{-2e^{-st}}{s}} - \frac{-2e^{-2s}}{s}$$

$$= \cancel{\frac{-2e^{-2s}}{s}} - \frac{e^{-2s}}{s^2} + \frac{1}{s^2} + \cancel{\frac{2e^{-2s}}{s}} = \frac{1}{s^2} - \frac{1}{s^2}e^{-2s} = U(s)$$

$$u(t) = \mathcal{L}^{-1}(U(s)) = U_r(t) - U_r(t-2)$$

$$Y(s) = H(s)\,U(s) = \frac{4}{2s+1}\left(\frac{1}{s^2} - \frac{1}{s^2}e^{-2s}\right)$$

$$= \frac{8e^{-2s} - 8}{s} - \frac{4e^{-2s} - 4}{s^2} - \frac{16e^{-2s} - 16}{2s+1}$$

$$= \frac{8}{s}e^{-2s} - \frac{8}{s} - \frac{4}{s^2}e^{-2s} + \frac{4}{s^2} - \frac{16}{2s+1}e^{-2s} + \frac{16}{2s+1}$$

$$y(t) = 8u_s(t-2) - 8u_s(t) - 4u_r(t-2) + 4u_r(t) - 8e^{-\frac{(t-2)}{2}}u_s(t-2) + 8e^{-t/2}u_s(t)$$