imp.ip Input impedance and admittance

1 We now introduce a generalization of the familiar impedance and admittance of electrical circuit analysis, in which system behavior can be expressed algebraically instead of differentially. We begin with generalized input impedance.

$Z_C = \frac{1}{j\omega C}$

$Z_C = \frac{1}{Cs}$

2 Consider a system with a source, as shown in Fig. ip.1. The source can be either an across- or a through-variable source. The ideal source specifies either $\mathcal{V}_{in}$ or $\mathcal{F}_{in}$, and the other variable depends on the system.

3 Let a source variables have Laplace transforms $\mathcal{V}_{in}(s)$ and $\mathcal{F}_{in}(s)$. We define the system's input impedance Z and input admittance Y to be the Laplace-domain ratios

input impedance
input admittance

$$Z(s) = \frac{\mathcal{V}_{in}(s)}{\mathcal{F}_{in}(s)} \quad \text{and} \quad Y(s) = \frac{\mathcal{F}_{in}(s)}{\mathcal{V}_{in}(s)}. \qquad (1)$$

Clearly,

$$\boxed{Y(s) = \frac{1}{Z(s)}}$$

Both Z and Y can be considered transfer functions: for a through-variable source $\mathcal{F}_{in}$, the impedance Z is the transfer function to across-variable $\mathcal{V}_{in}$; for an across-variable source $\mathcal{V}_{in}$, the admittance Y is the transfer function to through-variable $\mathcal{F}_{in}$. Often, however, we use the more common impedance Z to characterize systems with either type of source.

transfer functions

4 Note that Z and Y are system properties, not properties of the source. An impedance or admittance can characterize a system of interconnected elements, or a system of a single element, as the next section explores.

system properties

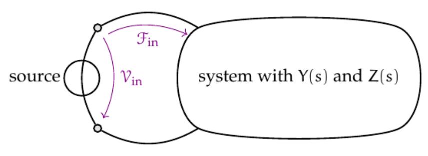


Figure ip.1:

Impedance of ideal passive elements

5 The impedance and admittance of a single, ideal, one-port element is defined from the Laplace transform of its elemental equation.

Generalized capacitors A generalized capacitor has elemental equation

generalized capacitor

$$\frac{d\mathcal{V}_C(t)}{dt} = \frac{1}{C}\mathcal{F}_C(t), \qquad (2)$$

the Laplace transform of which is

$$s\mathcal{V}_C(s) = \frac{1}{C}\mathcal{F}_C(s), \qquad (3)$$

which can be solved for impedance $Z_C = \mathcal{V}_C/\mathcal{F}_C$ and admittance $Y_C = \mathcal{F}_C/\mathcal{V}_C$:

$$\boxed{Z_C(s) = \frac{1}{Cs} \qquad Y_C(s) = Cs}$$

Generalized inductors A generalized inductor has elemental equation

generalized inductor

$$\frac{d\mathcal{F}_L(t)}{dt} = \frac{1}{L}\overset{V_L}{\cancel{\mathcal{F}_L}}(t), \qquad (4)$$

the Laplace transform of which is

$$s\mathcal{F}_L(s) = \frac{1}{L}\mathcal{V}_L(s), \qquad (5)$$

which can be solved for impedance $Z_L = \mathcal{V}_L/\mathcal{F}_L$ and admittance $Y_L = \mathcal{F}_L/\mathcal{V}_L$:

$$\boxed{Z_L(s) = Ls \qquad Y_L(s) = \frac{1}{Ls}}$$

Generalized resistors A generalized resistor has elemental equation

generalized resistor

$$\mathcal{V}_R(t) = \mathcal{F}_R(t)R, \qquad (6)$$

the Laplace transform of which is

$$\mathcal{V}_R(s) = \mathcal{F}_R(s)R, \qquad (7)$$

which can be solved for impedance $Z_R = \mathcal{V}_R/\mathcal{F}_R$ and admittance $Y_R = \mathcal{F}_R/\mathcal{V}_R$:

$$\boxed{Z_R = R \qquad Y_R = 1/R}$$

6 For a summary of the impedance of one-port elements, see Table els.1.

Impedance of interconnected elements

7 As with electrical circuits, impedances of linear graphs of interconnected elements can be combined in two primary ways: in parallel or in series.

8 Elements sharing the same through-variable are said to be in series connection. N elements connected in series Z_1 Z_2 ⋯ have equivalent impedance Z and admittance Y:

series

$$Z(s) = \sum_{i=1}^{N} Z_i(s) \quad \text{and} \quad Y(s) = 1 \Big/ \sum_{i=1}^{N} 1/Y_i(s) \qquad (8)$$

$N = 2$

$Y(s) = \frac{1}{\frac{1}{Y_1} + \frac{1}{Y_2}}$

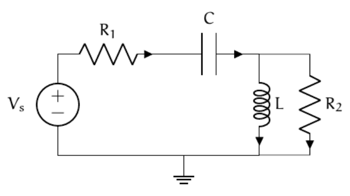
9 Conversely, elements sharing the same across-variable are said to be in parallel connection. N elements connected in parallel have equivalent impedance Z and admittance Y:

parallel

$$Z(s) = 1 \Big/ \sum_{i=1}^{N} 1/Z_i(s) \quad \text{and} \quad Y(s) = \sum_{i=1}^{N} Y_i(s). \qquad (9)$$

Example imp.ip–1 **re: input impedance of a simple circuit**

For the circuit shown, find the input impedance.



$Y_{LR_2} = Y_L + Y_{R_2}$

$Z_{LR_2} = \frac{1}{Y_L + Y_{R_2}} = \frac{1}{\frac{1}{Z_L} + \frac{1}{Z_{R_2}}}$

$Z = Z_{R_1} + Z_C + Z_{LR_2}$

$= Z_{R_1} + Z_C + \frac{Z_{R_2} Z_L}{Z_{R_2} + Z_L}$

$= \frac{Z_{R_2} Z_L}{Z_{R_2} + Z_L}$