imp.2port Impedance with two–port elements

1 The two types of energy transducing elements, transformers and gyrators, "reflect" or "transmit" impedance through themselves, such that they are "felt" on the other side.

transformers
gyrators

2 For a transformer, the elemental equations are

transformer

$$\mathcal{V}_2(t) = \mathcal{V}_1(t)/\mathrm{TF} \quad \text{and} \quad \mathcal{F}_2(t) = -\mathrm{TF}\mathcal{F}_1(t), \tag{1}$$

the Laplace transforms of which are

$$\mathcal{V}_2(s) = \mathcal{V}_1(s)/\mathrm{TF} \quad \text{and} \quad \mathcal{F}_2(s) = -\mathrm{TF}\mathcal{F}_1(s). \tag{2}$$

3 If, on the 2-side, the input impedance is Z_3, as in Fig. 2port.1, the equations of Eq. 2 are subject to the continuity and compatibility equations

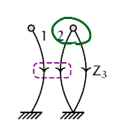


Figure 2port.1:

$$\mathcal{V}_2 = \mathcal{V}_3 \quad \text{and} \quad \mathcal{F}_2 = -\mathcal{F}_3. \tag{3}$$

Substituting these into Eq. 2 and solving for $\mathcal{V}_1$ and $\mathcal{F}_1$,

$$\mathcal{V}_1 = \mathrm{TF}\mathcal{V}_3 \quad \text{and} \quad \mathcal{F}_1 = \mathcal{F}_3/\mathrm{TF}. \tag{4}$$

The elemental equation for element 3 is $\mathcal{V}_3 = \mathcal{F}_3 Z_3$, which can be substituted into the through-variable equation to yield

$$\boxed{F_1 = \frac{1}{Z_3 TF} V_3}$$

4 Working our way back from $\mathcal{V}_3$ to $\mathcal{V}_1$, we apply the compatibility equation $\mathcal{V}_2 = \mathcal{V}_3$ and the elemental equation $\mathcal{V}_2 = \mathcal{V}_1/\mathrm{TF}$, as follows:

effective input impedance

$$\boxed{\begin{aligned} F_1 &= \frac{1}{Z_3 TF} V_2 \\ &= \frac{1}{Z_3 TF^2} V_1 \end{aligned}}$$

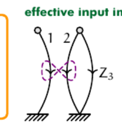


Figure 2port.2:

Solving for the effective input impedance Z_1,

gyrator

$$\begin{aligned} Z_1 &\equiv \frac{\mathcal{V}_1(s)}{\mathcal{F}_1(s)} && (5) \\ &= \mathrm{TF}^2 Z_3. && (6) \end{aligned}$$

5 For a gyrator with gyrator modulus GY, in the configuration shown in Fig. 2port.2, a similar derivation yields the effective input impedance Z_1,

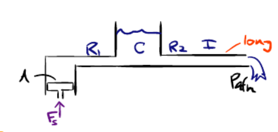
effective input impedance

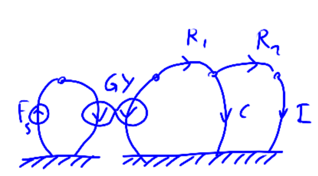
$$Z_1 = \mathrm{GY}^2/Z_3. \tag{7}$$

Example imp.2port–1

re: input impedance of fluid system with transducer

Draw a linear graph of the fluid system. What is the input impedance for an input force to the piston?





R_1	R_1
R_2	R_2
C	$1/Cs$
I	Is
GY	$-1/A$

$$Z_{R_2 I} = Z_{R_2} + Z_I$$

$$Z_{R_2 I C} = \frac{1}{\frac{1}{Z_{R_2} + Z_I} + \frac{1}{Z_C}}$$

$$Z_3 = Z_{R_1} + \frac{1}{\frac{1}{Z_{R_2} + Z_I} + \frac{1}{Z_C}} = R_1 + \frac{1}{\frac{1}{R_2 + Is} + Cs} = R_1 + \frac{R_2 + Is}{1 + R_2 Cs + CIs^2} = \frac{R_1 + R_1 R_2 Cs + R_1 CIs^2 + R_2 + Is}{1 + R_2 Cs + CIs^2}$$

$$Z_1 = GY^2 \, 1/Z_3 = \frac{1}{A^2} \frac{1 + R_2 Cs + ICs^2}{R_1 R_2 Cs + R_1 + R_1 CIs^2 + R_2 + Is} = \frac{V_s}{F_s}$$