# imp.divide The divider method

1 In Electronics, we developed the useful voltage divider formula for quickly analyzing how voltage divides among series electronic impedances. This can be considered a special case of a more general across-variable divider equation for any elements described by an impedance. After developing the across-variable divider, we also introduce the through-variable divider, which divides an input through-variable among parallel elements.

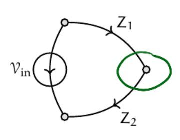



**Figure divide.1:** the two-element across-variable divider.

**across−variable divider**

$$V_o = \frac{R_2}{R_1+R_2} V_{in} \qquad \frac{V_o}{V_{in}} = \frac{R_2}{R_1+R_2}$$

### Across-variable dividers

2 First, we develop the solution for the two-element across-variable divider shown in Figure divide.1. We choose the across-variable across $Z_2$ as the output. The analysis follows the impedance method of Lecture imp.tf, solving for $\mathcal{V}_2$.

1. Derive four independent equations.
   a) The normal tree is chosen to consist of $\mathcal{V}_{in}$ and $Z_2$.
   b) The elemental equations are

   $$V_1 = F_1 Z_1$$
   $$V_2 = F_2 Z_2$$

   c) The continuity equation is $F_1 = F_2$
   d) The compatibility equation is $V_1 = V_{in} - V_2$
2. Solve for the output $\mathcal{V}_2$. From the elemental equation for $Z_2$,

$$\begin{aligned}\mathcal{V}_2 &= \mathcal{F}_\infty Z_2 \\ &= \frac{\mathcal{V}_1}{Z_1} Z_2 \\ &= \frac{Z_2}{Z_1}(\mathcal{V}_{in} - \mathcal{V}_2) \quad \Rightarrow \\ \mathcal{V}_2 &= \frac{Z_2}{Z_1 + Z_2}\mathcal{V}_{in}.\end{aligned}$$

3 A similar analysis can be conducted for $n$ impedance elements.

**Equation 1 general across−variable divider**

For output across variable across $Z_k$ in series with n impedances with input $V_{in}$ is

$$V_k = \frac{Z_k}{Z_1 + \cdots + Z_k + \cdots + Z_n} V_{in}$$

### Through-variable dividers

4 By a similar process, we can analyze a network that divides a through-variable into $n$ parallel impedance elements.

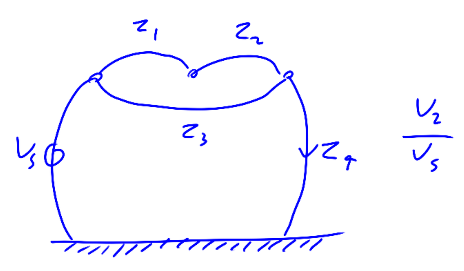
**Equation 2 general through−variable divider**

For output through variable through $Z_k$ in parallel with n impedances with input $F_s$

$$F_k = \frac{1/Z_k}{1/Z_1 + \cdots + 1/Z_k + \cdots + 1/Z_n} F_s$$

### Transfer functions using dividers

5 An excellent shortcut to deriving a transfer function is to use the across- and through-variable divider rules instead of solving the system of algebraic equations, as in Lec. imp.tf. An algorithm for this process is as follows.




1. Identify the element associated with an output variable $Y_i$. Call it the output element.
2. Identify the source associated with an input variable $U_j$. Set all other sources to zero.
3. Transform the network to be an across- or through-variable divider that includes the "bare" (uncombined) output element's output variable.[6]
   a) If necessary, form equivalent impedances of portions of the network, being sure to leave the output element's output variable alone.
   b) If necessary, transform the source à la Norton or Thévenin.
4. Apply the across- or through-variable divider equation.
5. If necessary, use the elemental equation of the output element to trade output across- and through-variables.
6. If necessary, use the source transformation equation of the input to trade input across- and through-variables.
7. Divide both sides by the input variable.

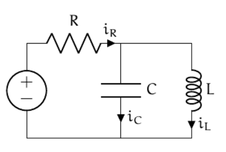
6. In other words, if the across-variable of the output element is the output, do not combine it in series; if the through-variable is the output, do not combine it in parallel.

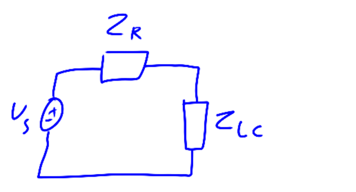
6 It turns out that, despite its many "if necessary" clauses, very often this "shortcut" is easier than the method of Lecture imp.tf for low-order systems if only a few transfer functions are of interest.

**Example imp.divide−1** **re: a circuit transfer function using a divider**

Given the circuit shown with voltage source $V_s$ and output $v_L$,

a. what is the transfer function $\frac{V_L}{V_s}$?
b. Without transforming the source, find the transfer function $\frac{I_L}{V_s}$.
c. Transforming the source, find $\frac{I_L}{V_s}$.




$$V_L = V_s \frac{Z_{LC}}{Z_R + Z_{LC}} \qquad Z_{LC} = \frac{1}{\frac{1}{Z_L} + \frac{1}{Z_C}}$$

$$= V_s \frac{Ls/(1+LCs^2)}{R + \frac{Ls}{1+LCs^2}} \cdot \frac{1+LCs^2}{1+LCs^2} \qquad = \frac{1}{\frac{1}{Ls} + Cs} \cdot \frac{Ls}{Ls}$$

$$= V_s \frac{Ls}{R + LCRs^2 + Ls} \qquad = \frac{Ls}{1+LCs^2}$$

$$\boxed{\frac{V_L}{V_s} = \frac{Ls}{R + LCRs^2 + Ls}}$$

$$V_L = V_s \frac{Ls}{R + LCRs^2 + Ls} \qquad V_L = Z_L I_L$$

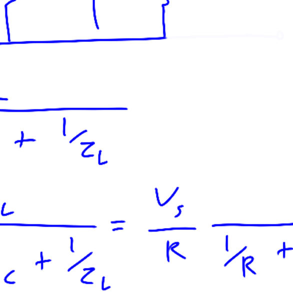
$$Z_L I_L = V_s \frac{Ls}{R + LCRs^2 + Ls}$$

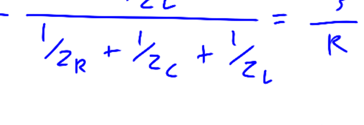
$$I_L = V_s \frac{1}{Ls} \frac{Ls}{R + LCRs^2 + Ls}$$

$$I_L = V_s \frac{1}{R + LCRs^2 + Ls}$$

$$\boxed{\frac{I_L}{V_s} = \frac{1}{R + LCRs^2 + Ls}}$$

$$Z_e = R$$
$$I_e = \frac{V_s}{R}$$




$$I_L = I_e \frac{1/Z_L}{1/Z_R + 1/Z_C + 1/Z_L}$$

$$= \frac{V_s}{R} \frac{1/Z_L}{1/Z_R + 1/Z_C + 1/Z_L} = \frac{V_s}{R} \frac{1/Ls}{1/R + Cs + 1/Ls}$$

$$= \frac{V_s}{R} \frac{1}{Ls/R + LCs^2 + 1}$$

$$I_L = V_s \frac{1}{Ls + LCRs^2 + R}$$

$$\boxed{\frac{I_L}{V_s} = \frac{1}{Ls + LCRs^2 + R}}$$