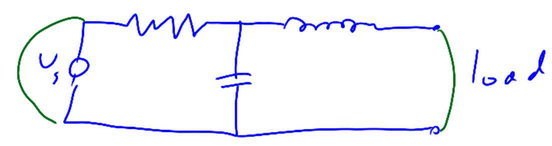
imp.eqiv Norton and Thévenin theorems

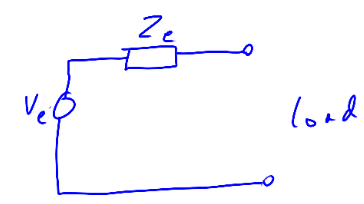
1 The following remarkable theorem has been proven.[5]

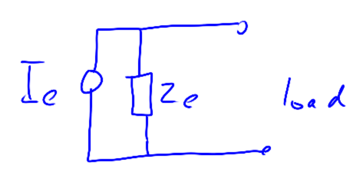
Theorem imp.1: generalized Thévenin's theorem

Given a linear network of across-variable sources, through-variable sources, and impedances, the behavior at the network's output nodes can be reproduced exactly by a single across-variable source $\mathcal{V}_e$ in series with an impedance Z_e.

5. This lecture is intentionally strongly paralleled in our Electronics lecture on Norton's and Thévenin's theorems.







2 The equivalent linear network has two quantities to determine: $\mathcal{V}_e$ and Z_e.

Determining Z_e

3 The equivalent impedance Z_e of a network is the impedance between the output nodes with all inputs set to zero. Setting an across-variable source to zero means the across-variable on both its terminals are equal, which is equivalent to treating them as the same node. Setting a through-variable source to zero means the through-variable through it is zero, which is equivalent to treating its nodes as disconnected.

equivalent impedance Z_e

Determining $\mathcal{V}_e$

4 The equivalent across-variable source $\mathcal{V}_e$ is the across-variable at the output nodes of the network when they are left open (disconnected from a load). Determining this value typically requires some analysis with the elemental, continuity, and compatibility equations (preferably via impedance methods).

equivalent across−variable source $\mathcal{V}_e$

Norton's theorem

5 Similarly, the following remarkable theorem has been proven.

Theorem imp.2: generalized Norton's theorem

Given a linear network of across-variable sources, through-variable sources, and impedances, the behavior at the network's output nodes can be reproduced exactly by a single through-variable source $\mathcal{F}_e$ in parallel with an impedance Z_e.

6 The equivalent network has two quantities to determine: $\mathcal{F}_e$ and Z_e. The equivalent impedance Z_e is identical to that of Thévenin's theorem, which leaves the equivalent through-variable source $\mathcal{F}_e$ to be determined.

Determining $\mathcal{F}_e$

7 The equivalent through-variable source $\mathcal{F}_e$ is the through-variable through the output terminals of the network when they are shorted (collapsed to a single node). Determining this value typically requires some analysis with elemental, continuity, and compatibility equations (preferably via impedance methods).

equivalent through−variable source $\mathcal{F}_e$

Converting between Thévenin and Norton equivalents

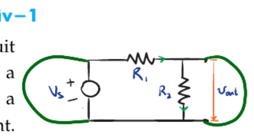
8 There is an equivalence between the two equivalent network models that allows one to convert from one to another with ease. The equivalent impedance Z_e is identical in each and provides the following equation for converting between the two representations:

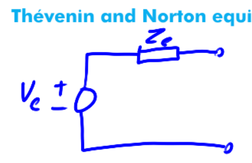
Equation 1 converting between Thévenin and Norton equivalents

$$V_e = Z_e F_e$$

Example imp.eqiv−1 re: **Thévenin and Norton equivalents**

For the circuit shown, find a Thévenin and a Norton equivalent.



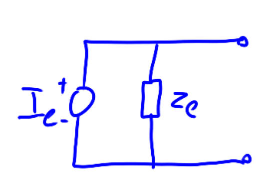


$$Z_e = \frac{1}{1/R_1 + 1/R_2} = \frac{R_1 R_2}{R_1 + R_2}$$

$$V_e = \frac{R_2}{R_1 + R_2} V_s$$

$$I_e = \frac{V_s}{R_1}$$

$$FZ = V$$



$$V_e = I_e Z_e$$

$$\frac{R_2}{R_1 + R_2} V_s = \frac{V_s}{\cancel{R_1}} \frac{\cancel{R_1} R_2}{R_1 + R_2}$$

$$= \frac{R_2}{R_1 + R_2} V_s \quad ✓$$