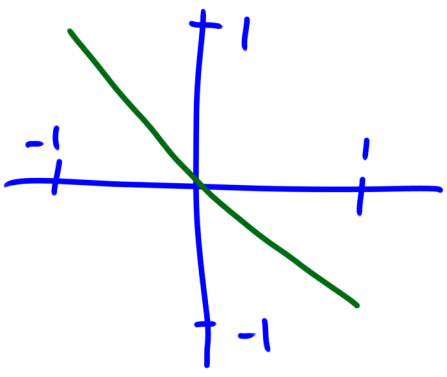

Fourier series

$T = 2$

$f(t) = -t$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(\omega_n t)\,dt$$

$$= \frac{2}{2}\int_{-1}^{1} -t\cos(\omega_n t)\,dt$$

$$= -\int_{-1}^{1} t\cos(\omega_n t)\,dt$$

$$\int u\,dv = uv - \int v\,du$$

$u = t$ $\quad dv = \cos(\omega_n t)\,dt$

$du = dt$ $\quad v = \dfrac{\sin(\omega_n t)}{\omega_n}$

$$= -\left(t\,\frac{\sin(\omega_n t)}{\omega_n} - \int_{-1}^{1}\frac{\sin(\omega_n t)}{\omega_n}\,dt\right)$$

$$= -\left(t\,\frac{\sin(\omega_n t)}{\omega_n} + \frac{\cos(\omega_n t)}{\omega_n^2}\Big|_{-1}^{1}\right)$$

$$= -\left(\frac{\sin(\omega_n)}{\omega_n} + \frac{\cos(\omega_n)}{\omega_n^2} - (-1)\,\frac{\sin(-\omega_n)}{\omega_n} - \frac{\cos(-\omega_n)}{\omega_n^2}\right)$$

$a_n = 0$

$$b_n = \frac{2}{2}\int_{-1}^{1} -t\sin(\omega_n t)\,dt = -\int_{-1}^{1} t\sin(\omega_n t)\,dt$$

$u = t$ $\quad dv = \sin(\omega_n t)\,dt$

$du = dt$ $\quad v = \dfrac{-\cos(\omega_n t)}{\omega_n}$

$$b_n = -\left(t\,\frac{-\cos(\omega_n t)}{\omega_n} - \int_{-1}^{1}\frac{-\cos(\omega_n t)}{\omega_n}\,dt\right)$$

$$= t\,\frac{\cos(\omega_n t)}{\omega_n} - \int_{-1}^{1}\frac{\cos(\omega_n t)}{\omega_n}$$

$$= t\,\frac{\cos(\omega_n t)}{\omega_n} - \frac{\sin(\omega_n t)}{\omega_n^2}\Big|_{-1}^{1}$$

$$= \frac{\cos(\omega_n)}{\omega_n} - \frac{\sin(\omega_n)}{\omega_n^2} - (-1)\,\frac{\cos(-\omega_n)}{\omega_n} + \frac{\sin(-\omega_n)}{\omega_n^2}$$

$$b_n = \boxed{\frac{2\cos(\omega_n)}{\omega_n} - \frac{2\sin(\omega_n)}{\omega_n^2}}$$

$$\omega_n = \frac{2\pi n}{T} = \frac{2\pi n}{2} = \pi n$$

$$f(t) \approx b_1\sin(\omega_1 t) + b_2\sin(\omega_2 t) + \cdots$$

$$= \frac{-2}{\pi}\sin(\pi t) + \frac{1}{\pi}\sin(2\pi t) - \frac{2}{3\pi}\sin(3\pi t) + \cdots$$