ME 370 – System Dynamics and Control

Midterm Exam 2

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Directions: take-home, open notes, open book. Use your own paper, work neatly, and clearly mark your answers. MATLAB and other programming languages may not be used. Partial credit may be given. Submit as a single PDF file.

Contents

Problem maximization

List three ways that a system's transfer function can be determined. ______/10 p.

linear graph → SS → TF

impedance methods

elemental eq'n algebra → IO ODE → TF

impulse response

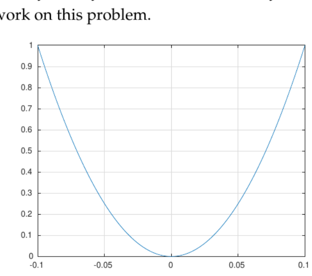
Problem suprenum

Given a periodic function $f(t) = 100t^2$ with a period $T = 0.2$ centered about the origin, also shown below: ______/25 p.

a calculate the first 5 components of the fourier series, and

b write the first 5 terms of the fourier series representation of the function.

Note: you may use MATLAB to check your work on this problem.



$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(\omega_n t)\,dt$$

$$= \frac{2}{0.2}\int_{-0.1}^{0.1} 100t^2\cos(\omega_n t)\,dt = 500\int_{-0.1}^{0.1} t^2\cos(\omega_n t)\,dt$$

$u = t^2 \qquad dv = \cos(\omega_n t)\,dt$

$du = 2t\,dt \qquad v = \frac{\sin(\omega_n t)}{\omega_n}$

$$= 1000\left(t^2\frac{\sin(\omega_n t)}{\omega_n} - \int_{-0.1}^{0.1}\frac{\sin(\omega_n t)}{\omega_n}2t\,dt\right) = 500\left(t^2\frac{\sin(\omega_n t)}{\omega_n} - \frac{2}{\omega_n}\int_{-0.1}^{0.1} t\sin(\omega_n t)\,dt\right)$$

$u = t \qquad dv = \sin(\omega_n t)$

$du = dt \qquad v = \frac{-\cos(\omega_n t)}{\omega_n}$

$$= 1000\left(t^2\frac{\sin(\omega_n t)}{\omega_n} - \frac{2}{\omega_n}\left(t\frac{-\cos(\omega_n t)}{\omega_n} - \int_{-0.1}^{0.1}\frac{-\cos(\omega_n t)}{\omega_n}\,dt\right)\right)$$

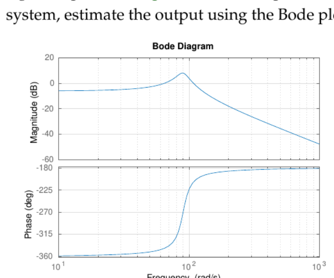
$$= 1000\left(t^2\frac{\sin(\omega_n t)}{\omega_n} - \frac{2}{\omega_n}\left(t\frac{-\cos(\omega_n t)}{\omega_n} + \frac{\sin(\omega_n t)}{\omega_n^2}\right)\right)\Bigg|_{-0.1}^{0.1}$$

$$= 1000\left(\frac{1}{100}\frac{\sin(\omega_n/10)}{\omega_n} - \frac{2}{\omega_n}\left(\frac{-\cos(\omega_n/10)}{10\omega_n} + \frac{\sin(\omega_n/10)}{\omega_n^2}\right)\right)$$

$$-1000\left(\frac{1}{100}\frac{\sin(-\omega_n/10)}{\omega_n} - \frac{2}{\omega_n}\left(\frac{\cos(-\omega_n/10)}{10\omega_n} + \frac{\sin(-\omega_n/10)}{\omega_n^2}\right)\right)$$

Problem bigish

The Bode plot for a system is shown below. Using the Fourier series representation of the signal in problem suprenum as an input to this system, estimate the output using the Bode plot. ______/15 p.



$$= 2000\left(\frac{1}{100}\frac{\sin(\omega_n/10)}{\omega_n} - \frac{2}{\omega_n}\left(\frac{-\cos(\omega_n/10)}{10\omega_n} + \frac{\sin(\omega_n/10)}{\omega_n^2}\right)\right)$$

$$a_n = \left(\frac{20}{\omega_n} - \frac{4000}{\omega_n^3}\right)\sin(\omega_n/10) + \frac{400}{\omega_n^2}\cos(\omega_n/10)$$

$$\omega_n = \frac{2\pi n}{T} = 10\pi n$$

$$a_n = \cancel{\left(\frac{2}{\pi n} - \frac{4}{\pi^3 n^3}\right)\sin(\pi n)} + \frac{4}{\pi^2 n^2}\cos(\pi n)$$

$$= \frac{4}{\pi^2 n^2}\cos(\pi n) \qquad b_n = 0$$

$a_1 = \frac{-4}{\pi^2}$, $a_2 = \frac{1}{\pi^2}$, $a_3 = \frac{-4}{9\pi^2}$, $a_4 = \frac{1}{4\pi^2}$, $a_5 = \frac{-4}{25\pi^2}$

$\omega_1 = 10\pi$, $\omega_2 = 20\pi$, $\omega_3 = 30\pi$, $\omega_4 = 40\pi$, $\omega_5 = 50\pi$

$$f(t) \approx \frac{-4}{\pi^2}\cos(10\pi t) + \frac{1}{\pi^2}\cos(20\pi t) - \frac{4}{9\pi^2}\cos(30\pi t) + \frac{1}{4\pi^2}\cos(40\pi t) - \frac{4}{25\pi^2}\cos(50\pi t)$$

$\angle H(j31) = -350° = -6.1$

$\angle H(j63) = -340° = -5.9$

$\angle H(j94) = -240° = -4.2$

$\angle H(j126) = -200° = -3.5$

$\angle H(j157) = -195° = -3.4$

$|H(j31)| = -4\,dB = 0.6$

$|H(j63)| = 0\,dB = 1$

$|H(j94)| = 6\,dB = 2$

$|H(j126)| = -7\,dB = 0.4$

$|H(j157)| = -10\,dB = 0.3$

$dB = 20\log_{10} mag$

$mag = 10^{dB/20}$

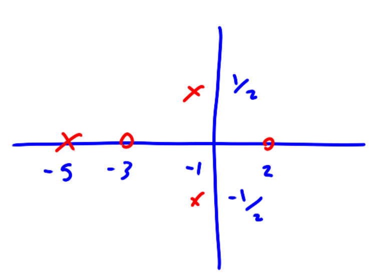
$$0.6\frac{-4}{\pi^2}\cos(10\pi t - 6.1) + \frac{1}{\pi^2}\cos(20\pi t - 5.9) - 2\frac{4}{9\pi^2}\cos(30\pi t - 4.2) + 0.4\frac{1}{4\pi^2}\cos(40\pi t - 3.5) - 0.3\frac{4}{25\pi^2}\cos(50\pi t - 3.4)$$

Problem enormous

Given a transfer function, ______/15 p.

$$H(s) = \frac{s^2 + s - 6}{(s^2 + 2s + 1.25)(s + 5)}$$

plot the location of the transfer function poles and zeros.



$s + 5 = 0$

$s = -5$

$s^2 + s - 6 = 0$

$(s - 2)(s + 3) = 0$

$s = 2, -3$

$s^2 + 2s + 1.25 = 0$

$$\frac{-2 \pm \sqrt{4 - 4(1.25)}}{2}$$

$$\frac{-2 \pm \sqrt{-1}}{2} = -1 \pm j/2$$

$s = -1 \pm j/2 = -1 + j/2,\ -1 - j/2$

$(-1 + j/2)^2 + 2(-1 + j/2) + 1.25 = 0$

$\cancel{1} - \cancel{j/2} - \cancel{j/2} - \cancel{1/4} - \cancel{2} + \cancel{j} + \cancel{1.25} = 0$

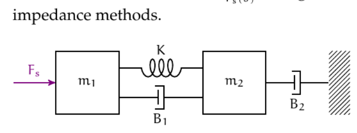
Problem immense

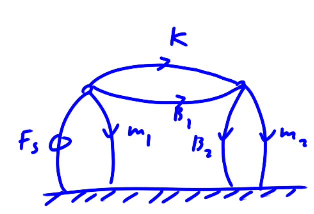
Given the system show below with a force input F_s: ______/35 p.

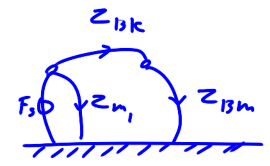
a draw a linear graph for the system,

b determine the system input impedance, and

c find the transfer function $\frac{V_{m_2}(s)}{F_s(s)}$ using impedance methods.







$$Z_{BK} = \frac{1}{\frac{1}{Z_{B_1}} + \frac{1}{Z_K}}$$

$$Z_{Bm} = \frac{1}{\frac{1}{Z_{B_2}} + \frac{1}{Z_{m_2}}}$$

$$Z_{in} = \frac{1}{\frac{1}{Z_{m_1}} + \frac{1}{Z_{BK} + Z_{Bm}}}$$

$$Z_{in} = \frac{V_s(s)}{F_s(s)}$$

$$Z_{in}F_s(s) = V_s(s)$$

$$V_{m_2} = V_s\frac{Z_{Bm}}{Z_{BK} + Z_{Bm}}$$

$$V_{m_2} = F_s Z_{in}\frac{Z_{Bm}}{Z_{BK} + Z_{Bm}}$$

$$\frac{V_{m_2}}{F_s} = Z_{in}\frac{Z_{Bm}}{Z_{BK} + Z_{Bm}}$$