intro.pidi An interactive PID controller design

1 In this lecture, we will build an interactive PID control design tool in Python. However, you need not install Python[2] to try the design tool: it is available at the following web page.

2. For more on Python, see python.org.

click to launch interactive page in browser

It may take a few minutes to load the Jupyter notebook.[3] Once it does, click Cell 〉Run All. This will run the Python code that comprises the remainder of this lecture. Scroll to the bottom of the webpage to interact with the PID gains that update the closed-loop step response plot!

3. For more on Jupyter, see jupyter.org.

2 For the unity feedback block diagram of Fig. pidi.1, we will design a PID controller $C(s)$. Design requirements are (a) less than 20 percent overshoot, (b) an initial peak in less than 0.2 seconds, and (c) zero steady-state error for a step response.

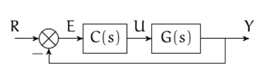


Figure pidi.1: a unity feedback control loop.

First, load some general-purpose Python packages.

3. Python code in this section was generated from a Jupyter notebook named pid_interactive_design_python.ipynb with a python3 kernel.

```
import numpy as np # for numerics
import sympy as sp # for symbolics
import control as c # the Control Systems module
import matplotlib as mpl # for plots
import matplotlib.pyplot as plt # also for plots
from IPython.display import display, Markdown, Latex
```

The following Python packages are specific for the interactive widget.

```
from ipywidgets import *
%matplotlib widget
```

Symbolic transfer functions

Let's investigate the transfer functions symbolically. We begin by defining the Laplace s and gain symbolic variables.

```
s,K_p,K_i,K_d = sp.symbols('s K_p K_i K_d')
```

We will design a PID controller for a plant with the following transfer function.

```
G_sym = 15000/(s**4+50*s**3+875*s**2+6250*s+15000)
display(G_sym)
```

$$\frac{15000}{s^4 + 50s^3 + 875s^2 + 6250s + 15000}$$

The controller has the following symbolic transfer function.

```
C_sym = K_p + K_i/s + K_d*s
display(C_sym)
```

$$K_d s + \frac{K_i}{s} + K_p$$

The closed-loop transfer function for the unity feedback system is as follows.

```
T_sym = sp.simplify(
  C_sym*G_sym/(1+C_sym*G_sym)
)
T_num, T_den = list( # for simplifying
  map(
    lambda x: sp.collect(x,s),
    sp.fraction(T_sym)
  )
)
T_sym = T_num/T_den
display(T_sym)
```

$$\frac{15000K_i + s\left(15000K_d s + 15000K_p\right)}{15000K_i + s\left(15000K_d s + 15000K_p + s^4 + 50s^3 + 875s^2 + 6250s + 15000\right)}$$

Symbolic to control transfer functions

The `control` package has objects of type `TransferFunction` that will be useful for simulation in the next section. We begin by defining a function to convert a symbolic transfer function to a `control` `TransferFunction` object.

```
def sym_to_tf(tf_sym,s_var):
    global s # changes s globally!
    S = s_var
    s = sp.symbols('s')
    tf_sym = tf_sym.subs(S,s)
    tf_str = str(tf_sym)
    s = c.TransferFunction.s
    ldict = {}
    exec('tf_out = '+tf_str,globals(),ldict)
    tf_out = ldict['tf_out']
    return tf_out
```

This isn't smooth, but it works. Note that `tf_sym` must have no symbolic variables besides `s_var`, the Laplace s. We can apply this to `G_sym`, then, but not yet `C_sym`.

```
type(sym_to_tf(G_sym,s))
```

```
control.xferfcn.TransferFunction
```

Defining the closed-loop function

We need to create a function that specifies the gains, substitutes them into the symbolic closed-loop transfer function, then converts it to a `control` package `TransferFunction` object via `sym_to_tf`.

```
def pid_CL_tf(CL_sym,Kp=0,Ki=0,Kd=0):
  sp.symbols('K_p K_i K_d')
  s = c.TransferFunction.s
  CL_subs = CL_sym.subs({K_p: Kp, K_i: Ki, K_d: Kd})
  return sym_to_tf(CL_subs,s)
```

For instance, we can let $K_p = 1$ and $K_i = K_d = 0$.

```
display(
  pid_CL_tf(T_sym,Kp=1)
)
```

$$\frac{1.5 \times 10^4}{s^4 + 50s^3 + 875s^2 + 6250s + 3 \times 10^4}$$

Step response

It is straightforward to use the `control` package's `step_response` function to get a step response for a single set of gains.

```
gains = {'Kp':2, 'Ki':1, 'Kd':0.1}
sys_CL = pid_CL_tf(T_sym,**gains)
t_step = np.linspace(0,3,200)
t_step,y_step = c.step_response(sys_CL, t_step)
```

Now let's plot it. The result is shown in Fig. pidi.2.

```
fig = plt.figure()
ax = fig.add_subplot(1, 1, 1)
line, = ax.plot(t_step, y_step)
plt.xlabel('time (s)')
plt.ylabel('step response')
plt.show()
```

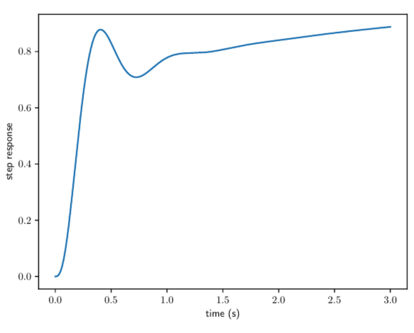


Figure pidi.2: step response with $K_p, K_i, K_d = 2, 1, 0.1$.

Interactive step response

The following essentially repeats the same process of

1. setting the PID gains with `pid_CL_tf`,
2. simulating with `step_response`, and
3. plotting the response.

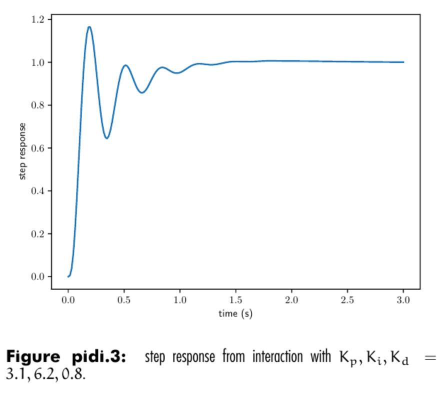
The caveat is that this happens with a GUI `interaction` callback function `update` that sets new gains (based on the GUI sliders), simulates, and replaces the old `line` on the plot. The final plot is shown in ??. It appears to meet our performance requirements.

```
%matplotlib widget
# simulate
t_step = np.linspace(0,3,200)
sys_CL = pid_CL_tf(T_sym,Kp=1)
t_step,y_step = c.step_response(sys_CL, t_step)

# initial plot
fig = plt.figure()
ax = fig.add_subplot(1, 1, 1)
line, = ax.plot(t_step, y_step)
plt.xlabel('time (s)')
plt.ylabel('step response')
plt.show()

# GUI callback function
def update(Kp = 1.0, Ki = 0.0, Kd = 0.0):
    global t_step, kp, ki, kd
    kp,ki,kd = Kp,Ki,Kd
    sys_CL = pid_CL_tf(T_sym,Kp=Kp,Ki=Ki,Kd=Kd)
    t_step,y_step = c.step_response(sys_CL, t_step)
    line.set_ydata(y_step)
    ax.relim()
    ax.autoscale_view()
    fig.canvas.draw_idle()
    plt.show()

# interaction definition
interact(
  update,
  Kp=(0.0,10.0),
  Ki=(0.0,20.0),
  Kd=(0.0,1.0)
);
```



The sliders appear as shown in Fig. pidi.4.

Figure pidi.3: step response from interaction with $K_p, K_i, K_d = 3.1, 6.2, 0.8$.

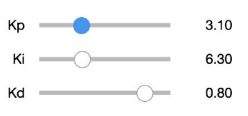


Figure pidi.4: this is how the sliders should look.