Calculators and MATLAB/Python may be used. Ones own notes, the textbook, and other resources provided by the instructor are allowed. No other resources shall be utilized. Communication about the exam with anyone other than the instructor during the exam period will be considered academic dishonesty. **All work on exams must be your own.** Anyone who does not abide by this policy will receive a **zero grade** on their exam. Work neatly and clearly mark your answers. Partial credit may be given.

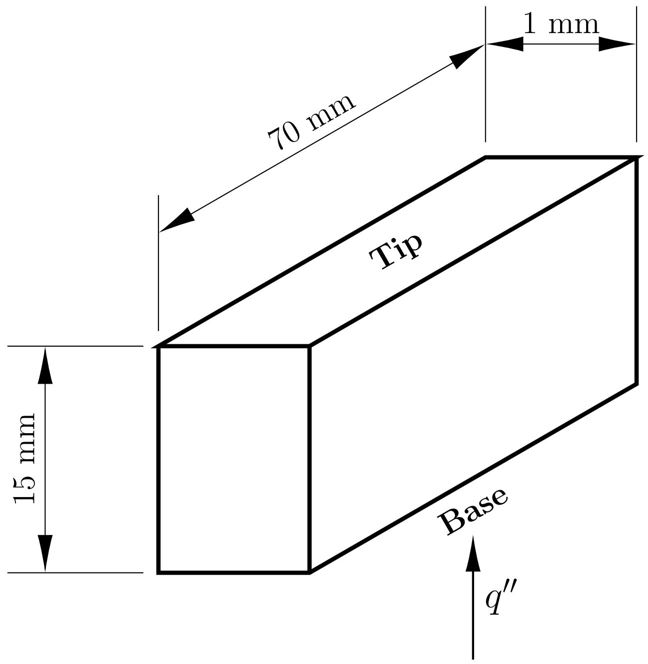
Name: ______________________________

Question:	1	2	3	4	5	Total
Points:	30	15	30	15	10	100
Score:						

1. (30 points) Dr. Devine is using his computer to solve for the steady state temperature distribution using a finite difference method. His computer processor is producing a heat flux of $q'' = 1.36 \times 10^4\ \frac{\mathrm{W}}{\mathrm{m}^2}$. One of the fins of the aluminum heatsink on the processor is shown below. Air blows over 5 sides of the fin (all sides except for the base) at a temperature, $T_\infty = 21\ °C$, and with a convection coefficient, $h = 50\ \frac{\mathrm{W}}{\mathrm{m}^2\,\mathrm{K}}$. Assuming steady state,

   (a) what is the temperature at the base of the fin, and

   (b) what is the temperature at the tip of the fin?



Solution: To solve this problem, we can consider it an extended surface. We are told that there is convection on 5 of the sides of the fin. This means that there is convection on the tip of the fin. Therefore the fin heat transfer rate is,

$$q_f = M \frac{\sinh(mL) + (h/mk)\cosh(mL)}{\cosh(mL) + (h/mk)\sinh(mL)}.$$

All the constants can be solved except for M. In order to do this, other constant must be calculated, fin perimiter $P = 2(0.001\,\mathrm{m} + 0.07\,\mathrm{m}) = 0.142\,\mathrm{m}$, fin cross-sectional area $A_c = 0.001\,\mathrm{m} \cdot 0.07\,\mathrm{m} = 7 \times 10^{-5}\,\mathrm{m}^2$, fin length $L = 0.015\,\mathrm{m}$, and

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{50\ \frac{\mathrm{W}}{\mathrm{m}^2\,\mathrm{K}}\ 0.142\,\mathrm{m}}{237\ \frac{\mathrm{W}}{\mathrm{m}\,\mathrm{K}}\ 7 \times 10^{-5}\,\mathrm{m}^2}} = 20.7\,\frac{1}{\mathrm{m}}.$$

Solving for M results in the equation,

$$
\begin{aligned}
M &= q'' A_c \frac{\cosh(mL) + (h/mk)\sinh(mL)}{\sinh(mL) + (h/mk)\cosh(mL)} \\
&= 1.36\times 10^{4}\,\frac{\mathrm{W}}{\mathrm{m}^2}\;7\times 10^{-7}\,\mathrm{m}^2\;\frac{\cosh(20.7\,\frac{1}{\mathrm{m}}\;0.015\,\mathrm{m}) + \left(\frac{50\,\frac{\mathrm{W}}{\mathrm{m}^2\,\mathrm{K}}}{20.7\,\frac{1}{\mathrm{m}}\;237\,\frac{\mathrm{W}}{\mathrm{m\,K}}}\right)\sinh(20.7\,\frac{1}{\mathrm{m}}\;0.015\,\mathrm{m})}{\sinh(20.7\,\frac{1}{\mathrm{m}}\;0.015\,\mathrm{m}) + \left(\frac{50\,\frac{\mathrm{W}}{\mathrm{m}^2\,\mathrm{K}}}{20.7\,\frac{1}{\mathrm{m}}\;237\,\frac{\mathrm{W}}{\mathrm{m\,K}}}\right)\cosh(20.7\,\frac{1}{\mathrm{m}}\;0.015\,\mathrm{m})} \\
&= 3.08\,\mathrm{W}.
\end{aligned}
$$

Using the equation $M = \sqrt{hPkA_c}\theta_b$,

$$
\begin{aligned}
\theta_b &= \frac{M}{\sqrt{hPkA_c}} \\
&= \frac{3.08\,\mathrm{W}}{\sqrt{50\,\frac{\mathrm{W}}{\mathrm{m}^2\,\mathrm{K}}\;0.142\,\mathrm{m}\;237\,\frac{\mathrm{W}}{\mathrm{m\,K}}\;7\times 10^{-5}\,\mathrm{m}^2}} \\
&= 8.96\,^\circ\mathrm{C}.
\end{aligned}
$$

Using the equation $\theta_b = T_b - T_\infty$,

$$
\begin{aligned}
T_b &= \theta_b + T_\infty \\
&= 8.96\,^\circ\mathrm{C} + 21\,^\circ\mathrm{C} \\
&= \boxed{30\,^\circ\mathrm{C}}.
\end{aligned}
$$

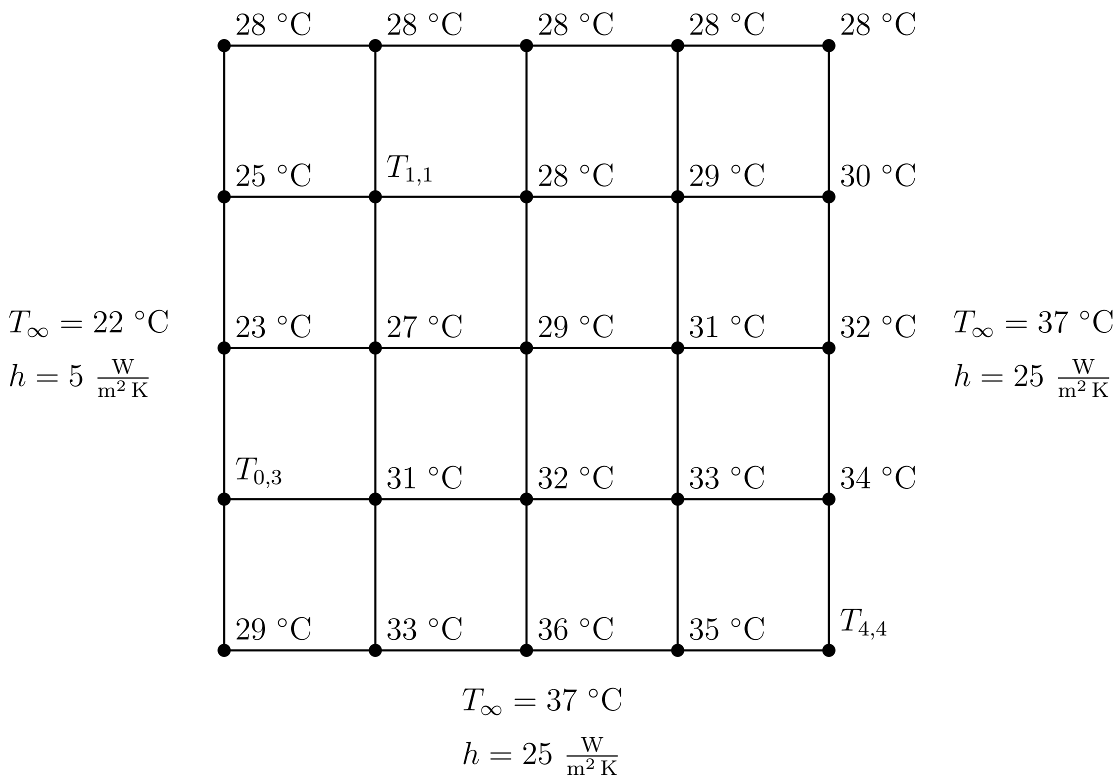
To find the temperature at the tip use the equation,

$$
\begin{aligned}
\frac{\theta}{\theta_b} &= \frac{\cosh(m(L-x)) + (h/mk)\sinh(m(L-x))}{\cosh(mL) + (h/mk)\sinh(mL)} \\
&= \frac{1}{\cosh(20.7\,\frac{1}{\mathrm{m}}\;0.015\,\mathrm{m}) + \left(\frac{50\,\frac{\mathrm{W}}{\mathrm{m}^2\,\mathrm{K}}}{20.7\,\frac{1}{\mathrm{m}}\;237\,\frac{\mathrm{W}}{\mathrm{m\,K}}}\right)\sinh(20.7\,\frac{1}{\mathrm{m}}\;0.015\,\mathrm{m})} \\
&= 0.95.
\end{aligned}
$$

This result can be used to calculate the temperature at the tip of the fin,

$$\begin{aligned}
\frac{\theta}{\theta_b} &= 0.95 \\
\theta &= 0.95\,\theta_b \\
T - T_\infty &= 0.95\,\theta_b \\
T &= 0.95\,\theta_b + T_\infty \\
&= 0.95\ 8.96\,^\circ\mathrm{C} + 21\,^\circ\mathrm{C} \\
&= \boxed{29.5\,^\circ\mathrm{C}}.
\end{aligned}$$

2. (15 points) After solving the temperature distribution from Problem 1, Dr. Devine's computer overheated. This caused it to lose the temperature at several nodes. Calculate the missing values. The material being simulated is oak hardwood, and the spacing of the grid is $\Delta x = 0.1$ m.



Solution: To solve for the temperature $T_{1,1}$, use the equation,

$$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} - 4T_{m,n} = 0.$$

Solving this equation for $T_{m,n}$ results in the equation,

$$T_{m,n} = \frac{T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n}}{4}.$$

Substituting values into the equation,

$$\begin{aligned} T_{1,1} &= \frac{27 + 28 + 28 + 25}{4} \\ &= \boxed{27\ ^\circ\text{C}}. \end{aligned}$$

To solve for $T_{0,3}$, use the equation,

$$2T_{m-1,n} + T_{m,n+1} + T_{m,n-1} + \frac{2h\Delta x}{k}T_\infty - 2\left(\frac{h\Delta x}{k} + 2\right)T_{m,n} = 0.$$

This equation was written for a right hand edge, but $T_{0,3}$ is on the left hand edge, so $T_{m-1,n}$ should be replaced with $T_{m+1,n}$. Utilizing this substitution, and solving for $T_{m,n}$ results in the equation,

$$\begin{aligned}
T_{m,n} &= \frac{2T_{m+1,n} + T_{m,n+1} + T_{m,n-1} + \frac{2h\Delta x}{k}T_\infty}{2\left(\frac{h\Delta x}{k} + 2\right)} \\
&= \frac{2\cdot 31 + 29 + 23 + \frac{2\cdot 5\cdot 0.1}{0.16}22}{2\left(\frac{5\cdot 0.1}{0.16} + 2\right)} \\
&= \boxed{24.54\ ^\circ\mathrm{C}}.
\end{aligned}$$

To solve for $T_{4,4}$, use the equation,

$$T_{m,n-1} + T_{m-1,n} + 2\frac{h\Delta x}{k}T_\infty - 2\left(\frac{h\Delta x}{k} + 1\right)T_{m,n} = 0.$$

This equation can also be solve for $T_{m,n}$,

$$\begin{aligned}
T_{m,n} &= \frac{T_{m,n-1} + T_{m-1,n} + 2\frac{h\Delta x}{k}T_\infty}{2\left(\frac{h\Delta x}{k} + 1\right)} \\
&= \frac{34 + 35 + 2\frac{25\cdot 0.1}{0.16}37}{2\left(\frac{25\cdot 0.1}{0.16} + 1\right)} \\
&= \boxed{36.85\ ^\circ\mathrm{C}}.
\end{aligned}$$

3. (30 points) With all the computer problems he is having, Dr. Devine is feeling hungry. He decides to get some food. He looks inside his (infinitely large) pantry and finds an (infinitely large) potato. This potato is strangely shaped, it is infinitely wide, infinitely deep, and is 60 mm thick. Dr. Devine places the potato in his (infinitely large) air fryer and sets it to 175 °C and waits for the minimum temperature in the potato to reach 100 °C. The potato is initially at 21 °C, the air fryer has a convection coefficient of 55 $\frac{\mathrm{W}}{\mathrm{m^2\,K}}$, and further details about the thermal properties of potatoes can be found in Appendix A.

   (a) How long will Dr. Devine have to wait until his potato is cooked?

   (b) Once the potato is cooked, what will the surface temperature of the potato be?

Solution: This problem can be considered a plane wall which has a temperature distribution,

$$\theta^* = C_1 \exp(-\zeta_1^2 Fo)\cos(\zeta_1 x^*).$$

The minimum temperature of the potato, when the surrounding temperature is hotter, will be in the middle, $x^* = x/L = 0$. The goal temperature of the potato sets the value for θ^*,

$$\begin{aligned}\theta^* &= \frac{T - T_\infty}{T_i - T_\infty} \\ &= \frac{100 - 175}{21 - 175} \\ &= 0.487.\end{aligned}$$

The final constants for this equation depend on the Biot number,

$$Bi = \frac{hL}{k} = \frac{55 \cdot 0.03}{0.56} = 2.98.$$

Using tables, the values for ζ_1 and C_1 can be found. The equation can now be solved to find the Fourier number,

$$\begin{aligned}Fo &= \frac{-1}{\zeta_1^2}\ln\left(\frac{\theta^*}{C_1}\right) \\ &= \frac{-1}{1.19^2}\ln\left(\frac{0.487}{1.21}\right) \\ &= 0.64.\end{aligned}$$

The time to cook the potato can now be found from the Fourier number, but the thermal diffusivity first must be found,

$$\alpha = \frac{k}{\rho c_p} = \frac{0.56}{1081 \cdot 3430} = 1.51 \times 10^{-7}\ \frac{\mathrm{m}^2}{\mathrm{s}}.$$

The Fourier number, $Fo = \alpha t/L^2$, can now be used to solve for t,

$$\begin{aligned} t &= \frac{L^2 Fo}{\alpha} \\ &= \frac{0.03^2 \cdot 0.64}{1.51 \times 10^{-7}} \\ &= 3831\ \mathrm{s} \\ &= \boxed{64\ \mathrm{min}}. \end{aligned}$$

To now find the temperature at the surface, $x^* = x/L = 1$, the same equation can be used.

$$\begin{aligned} \theta^* &= C_1 \exp(-\zeta_1^2 Fo) \cos(\zeta_1 x^*) \\ &= 1.21 \exp(-1.19^2 \cdot 0.64) \cos(1.19) \\ &= 0.18. \end{aligned}$$

From this value, the surface temperature can be found using the equation,

$$\theta^* = \frac{T - T_\infty}{T_i - T_\infty}.$$

Solved for T, the surface temperature is found as,

$$\begin{aligned} T &= \theta^*(T_i - T_\infty) + T_\infty \\ &= 0.18(21 - 175) + 175 \\ &= \boxed{147\ ^\circ\mathrm{C}}. \end{aligned}$$

4. (15 points) Annoyed at how long it is taking for his potato to cook, Dr. Devine looks in his pantry again and finds a more normal potato and places it in the microwave. The microwave can be considered to cause internal heat generation inside the potato, $\dot{q} = 8 \times 10^6 \ \frac{\mathrm{W}}{\mathrm{m}^3}$. If the potato is spherical with a radius of 35 mm, and there is no convection between the potato and the air in the microwave, how long will it take to cook this potato from the same initial temperature to the temperature listed in Problem 3?

Solution: Since there is no convection in this problem, the equations can be very simple. Starting with the spacific heat capacity, $c = 3430 \ \frac{\mathrm{J}}{\mathrm{kg\,K}}$, and multiplying this by the temperature change, $\Delta T = 100 - 21 = 79$ °C, results in the relationship, $\Delta T c = 271 \ \frac{\mathrm{J}}{\mathrm{kg}}$. This value can now be divided by the internal heat genreation rate,

$$\frac{\Delta T c}{\dot{q}} = 79 \text{ K } 3420 \ \frac{\mathrm{J}}{\mathrm{kg\,K}} \ \frac{1}{8 \times 10^6} \ \frac{\mathrm{m}^3}{\mathrm{W}} = 0.034 \ \frac{\mathrm{m}^3\,\mathrm{s}}{\mathrm{kg}},$$

since 1 J = 1 W s. This can be converted by to time by multiplying by density,

$$\frac{\Delta T c \rho}{\dot{q}} = 79 \text{ K } 3420 \ \frac{\mathrm{J}}{\mathrm{kg\,K}} \ \frac{1}{8 \times 10^6} \ \frac{\mathrm{m}^3}{\mathrm{W}} \ 1081 \ \frac{\mathrm{kg}}{\mathrm{m}^3} = \boxed{36.6 \text{ s}}.$$

5. (10 points) By now Dr. Devine is very hungry, but his potato from Problem 4 is too hot to eat. He decides to submerge his potato 50 mm deep in a very large tub of sour cream straight from the fridge at 5 °C. What is the rate of heat transfer from the potato to the sour cream?

   Solution: For this problem, the method of shape factors can be used. In this case, the shape factor is,

$$\begin{aligned} S &= \frac{2\pi D}{1 - \frac{D}{4z}} \\ &= \frac{2\pi 0.07}{1 - \frac{0.07}{4\cdot 0.05}} \\ &= 0.67 \text{ m.} \end{aligned}$$

   The heat transfer rate can now be found as,

$$\begin{aligned} q &= Sk(T_1 - T_2) \\ &= 0.67 \cdot 0.56(100 - 5) \\ &= \boxed{36 \text{ W}}. \end{aligned}$$

A Thermal Properties of Potatoes

Specific Heat Capacity $c = 3430 \, \frac{\mathrm{J}}{\mathrm{kg\,K}}$

Thermal Conductivity $k = 0.56 \, \frac{\mathrm{W}}{\mathrm{m\,K}}$

Density $\rho = 1081 \, \frac{\mathrm{kg}}{\mathrm{m}^3}$