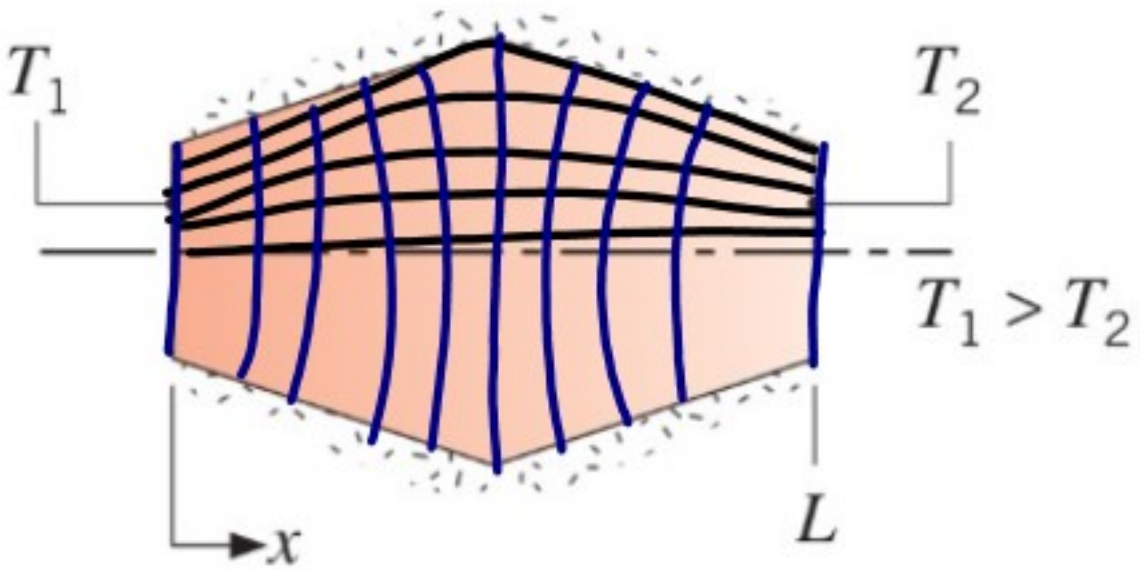
Fourier Law

$$q'' = -K \nabla T$$

isotherms

lines of constant temperature

Assume steady-state, one-dimensional conduction in the axisymmetric object below, which is insulated around its perimeter.



If the properties remain constant and no internal heat generation occurs, sketch the heat flux distribution, $q_x''(x)$, and the temperature distribution, $T(x)$. Explain the shapes of your curves. How do your curves depend on the thermal conductivity of the material?

Heat Diffusion

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$\dot{q}$ heat generation

if isotropic

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\nabla^2 T + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$K\nabla^2 T + \dot{q} = 0$ in steady state

Cylindrical Coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Spherical Coordinates

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(K\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{\dot{q}}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Boundary Conditions

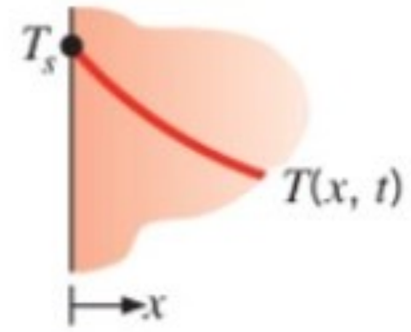
$T(x,t)$

Table 2.2

1. Constant surface temperature

$$T(0,t) = T_s \qquad (2.31)$$

Dirichlet

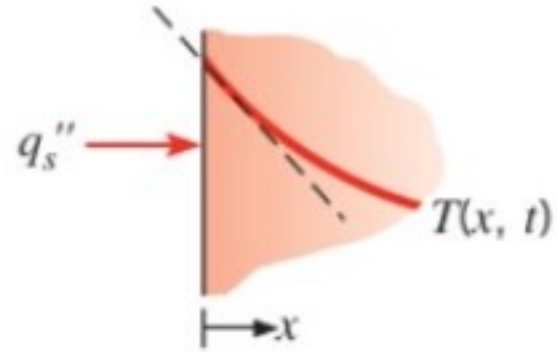


2. Constant surface heat flux

Neumann

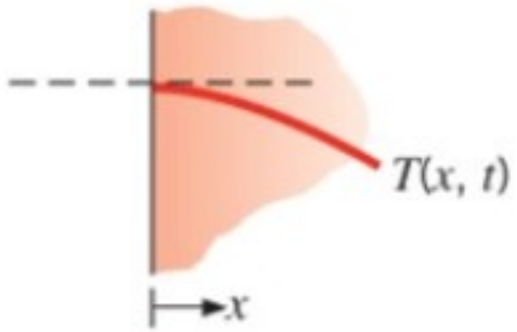
(a) Finite heat flux

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = q_s'' \qquad (2.32)$$



(b) Adiabatic or insulated surface

$$\frac{\partial T}{\partial x}\bigg|_{x=0} = 0 \qquad (2.33)$$



3. Convection surface condition

$q'' = h(T_\infty - T_s)$

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = h[T_\infty - T(0,t)] \qquad (2.34)$$