

1D Conduction



$$\nabla^2 T + \cancel{\frac{\dot{q}}{k}} = \frac{1}{\alpha} \cancel{\frac{\partial T}{\partial t}} \quad \text{steady state}$$

$$\nabla^2 T = 0$$

$$\frac{\partial^2 T}{\partial x^2} = 0$$

$$T(x) = c_1 x + c_2$$

$$T(0) = T_{s1} \quad c_1 \cdot 0 + c_2 = T_{s1}$$

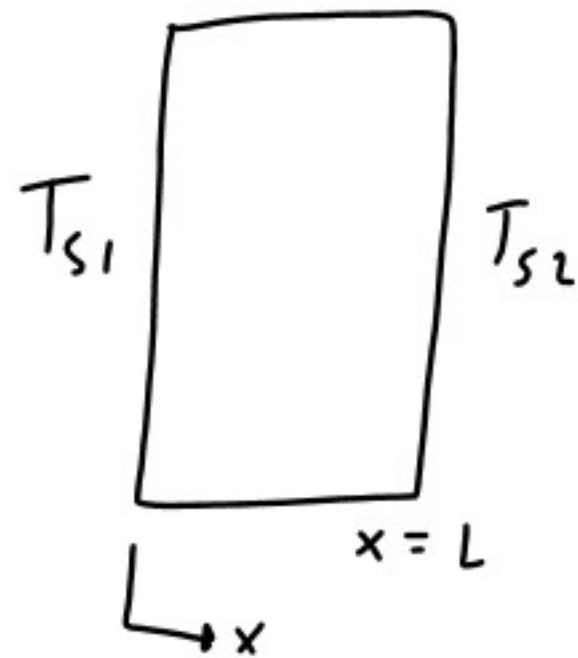
$$c_2 = T_{s1}$$

$$T(L) = T_{s2} \quad c_1 L + c_2 = T_{s2}$$

$$c_1 L + T_{s1} = T_{s2}$$

$$c_1 = \frac{T_{s2} - T_{s1}}{L}$$

$$T(x) = (T_{s2} - T_{s1}) \frac{x}{L} + T_{s1}$$



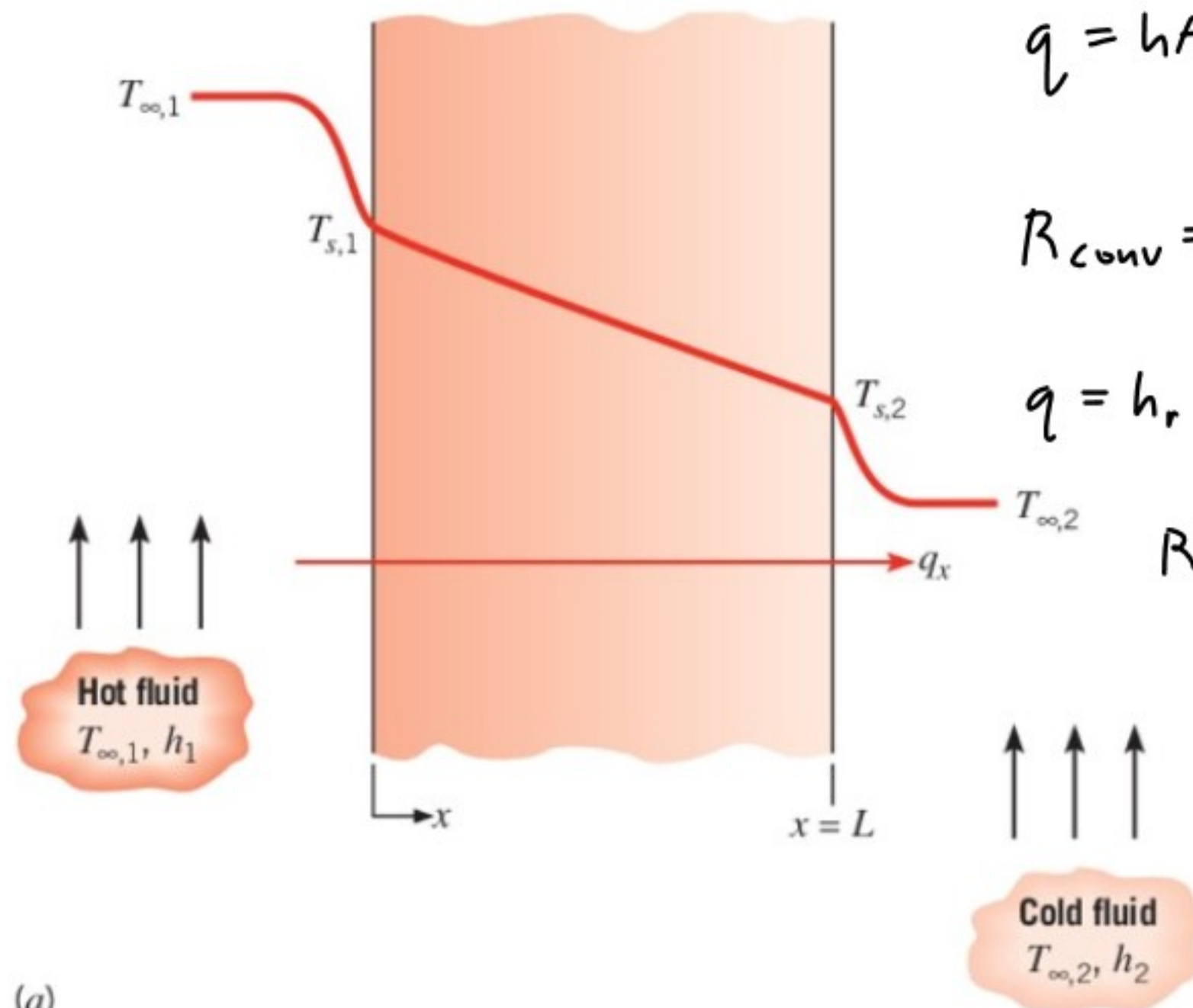
$$T(x) = (T_{s2} - T_{s1}) \frac{x}{L} + T_{s1}$$

$$\frac{\partial T}{\partial x} = \frac{T_{s2} - T_{s1}}{L}$$

$$\frac{\partial^2 T}{\partial x^2} = 0 \quad \checkmark$$

$$R \equiv \frac{\Delta T}{q}$$

$$R_{\text{cond}} = \frac{T_{s1} - T_{s2}}{q} = \frac{L}{kA}$$

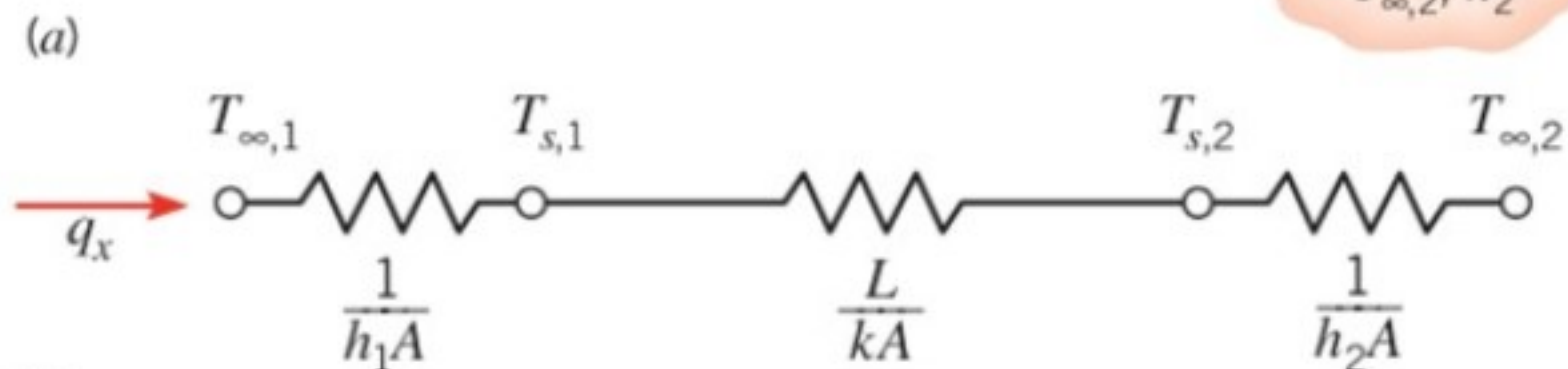


$$q = hA(T_s - T_{\infty})$$

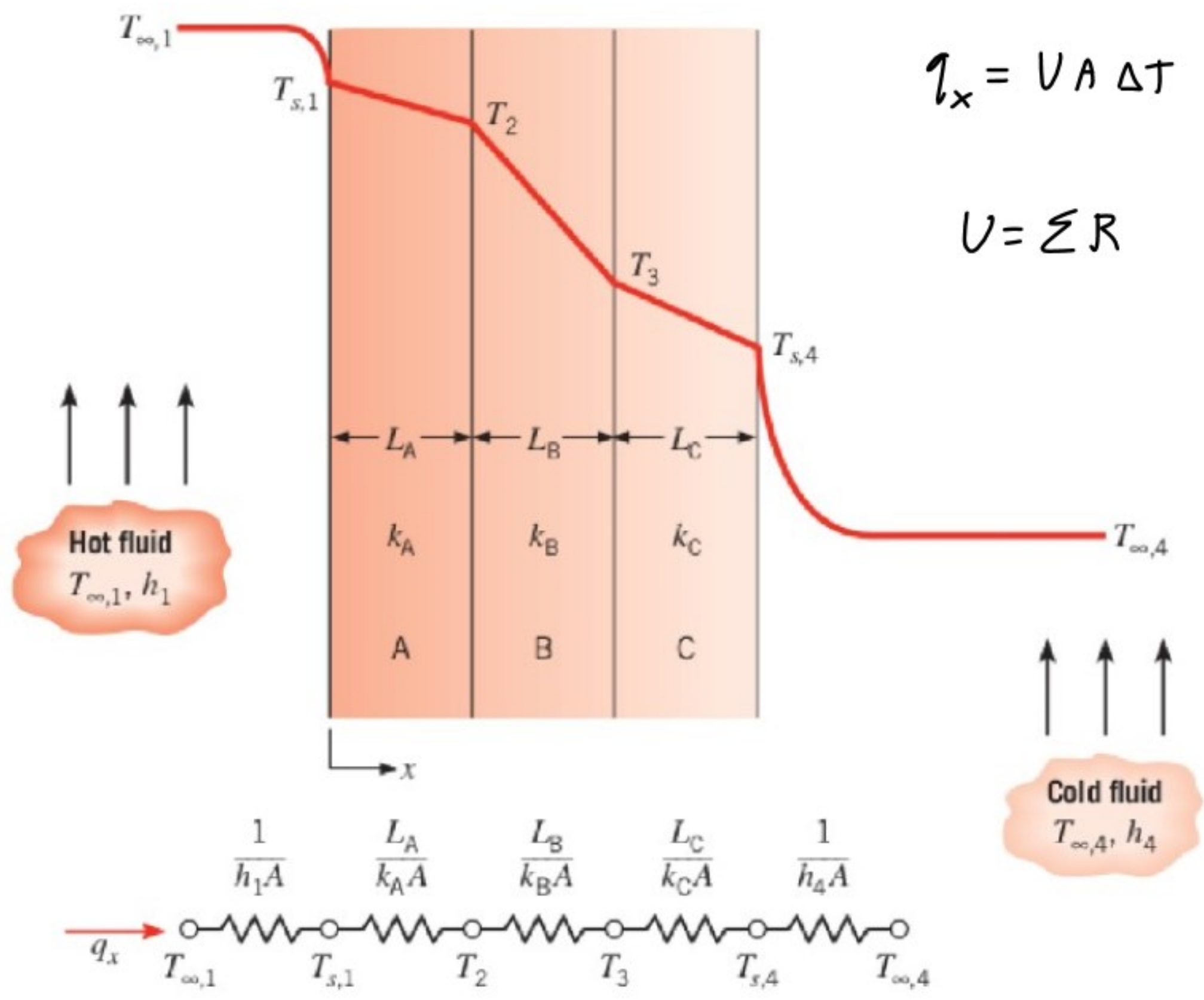
$$R_{\text{conv}} = \frac{T_s - T_{\infty}}{q} = \frac{1}{hA}$$

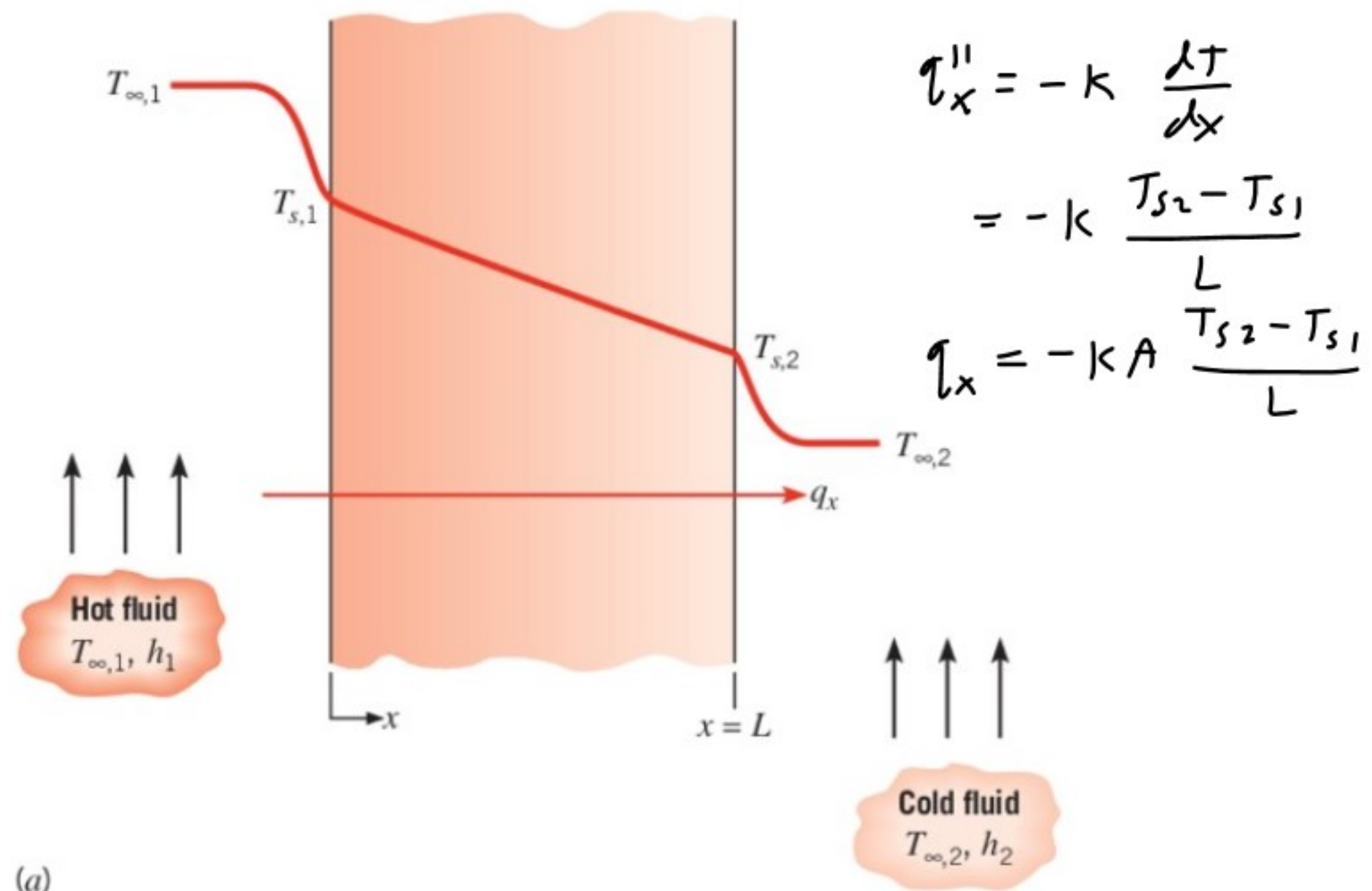
$$q = h_r A(T_s - T_{\text{sur}})$$

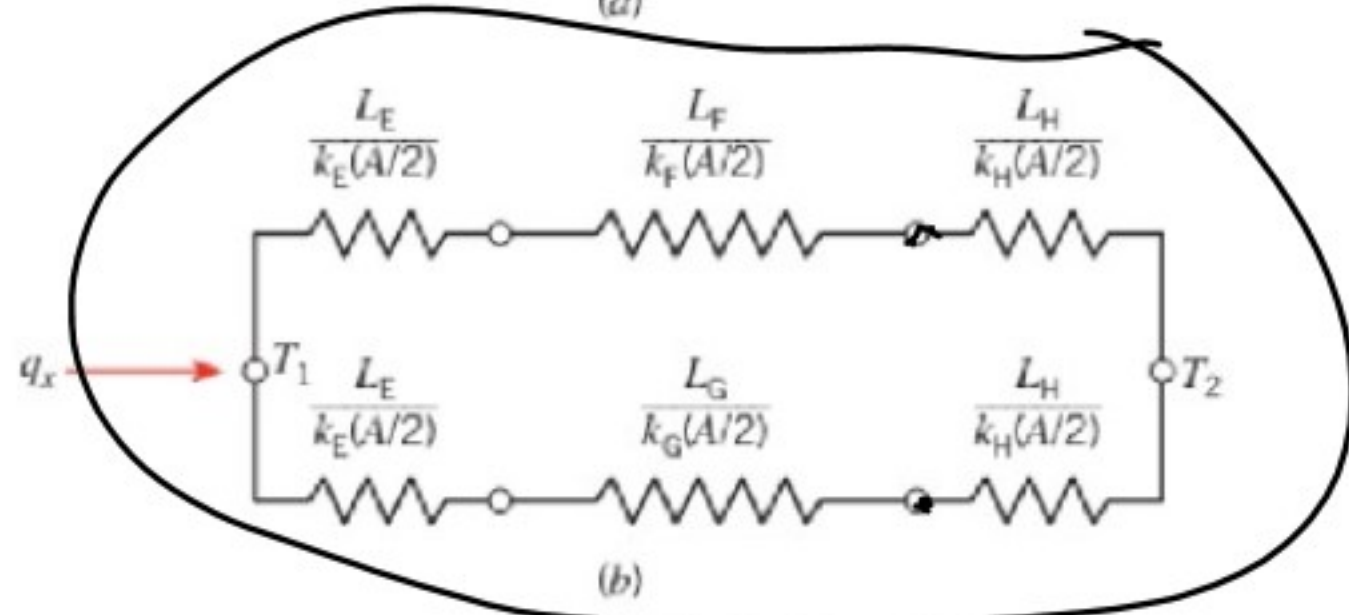
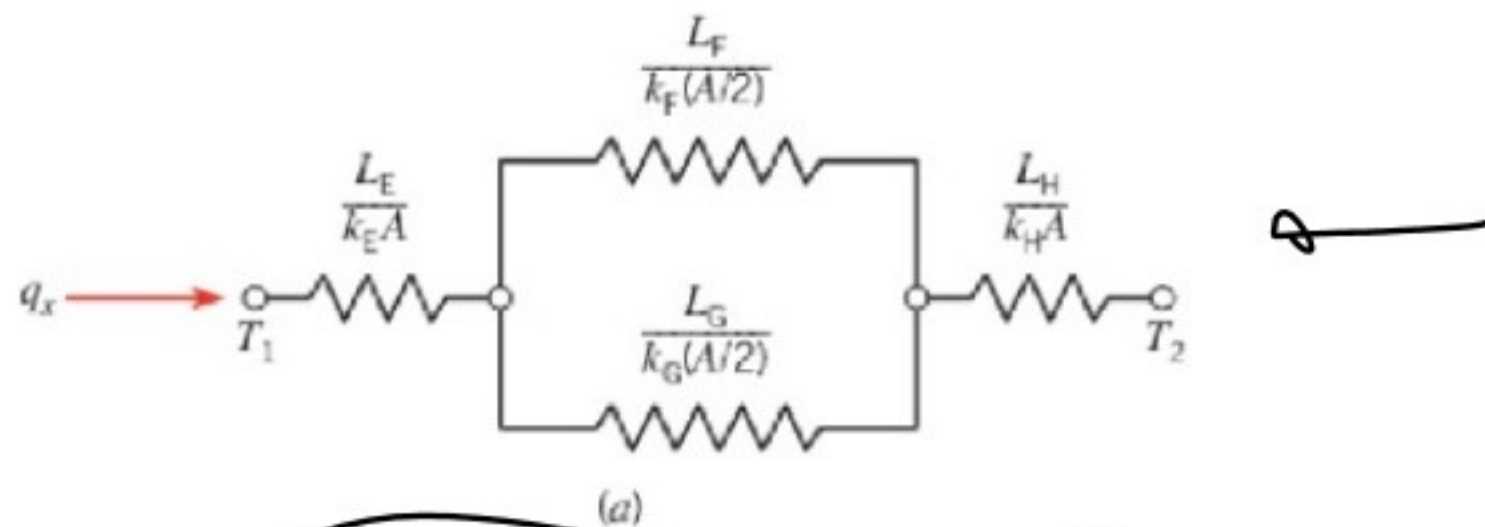
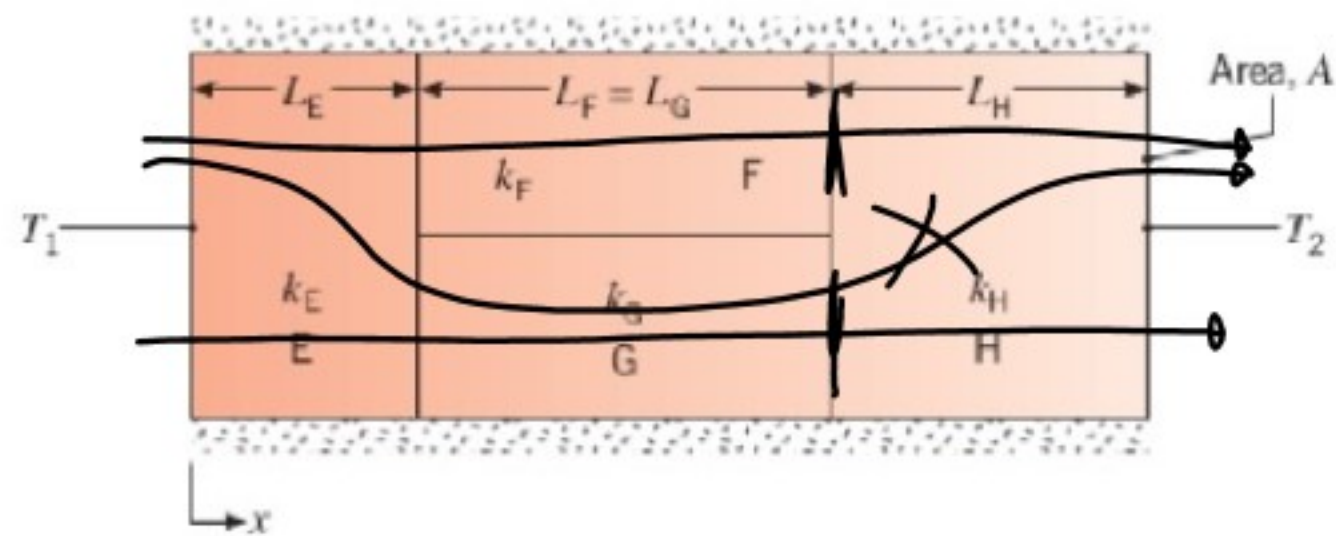
$$R = \frac{T_s - T_{\text{sur}}}{q} = \frac{1}{h_r A}$$



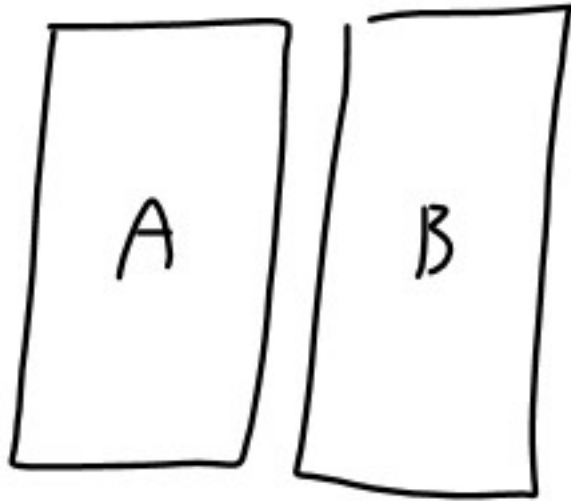
(b)







Contact Resistance



$$R_c = \frac{T_A - T_B}{q}$$

$$R_c'' = \frac{T_A - T_B}{q''}$$

$$q = -kA \frac{\Delta T}{L}$$

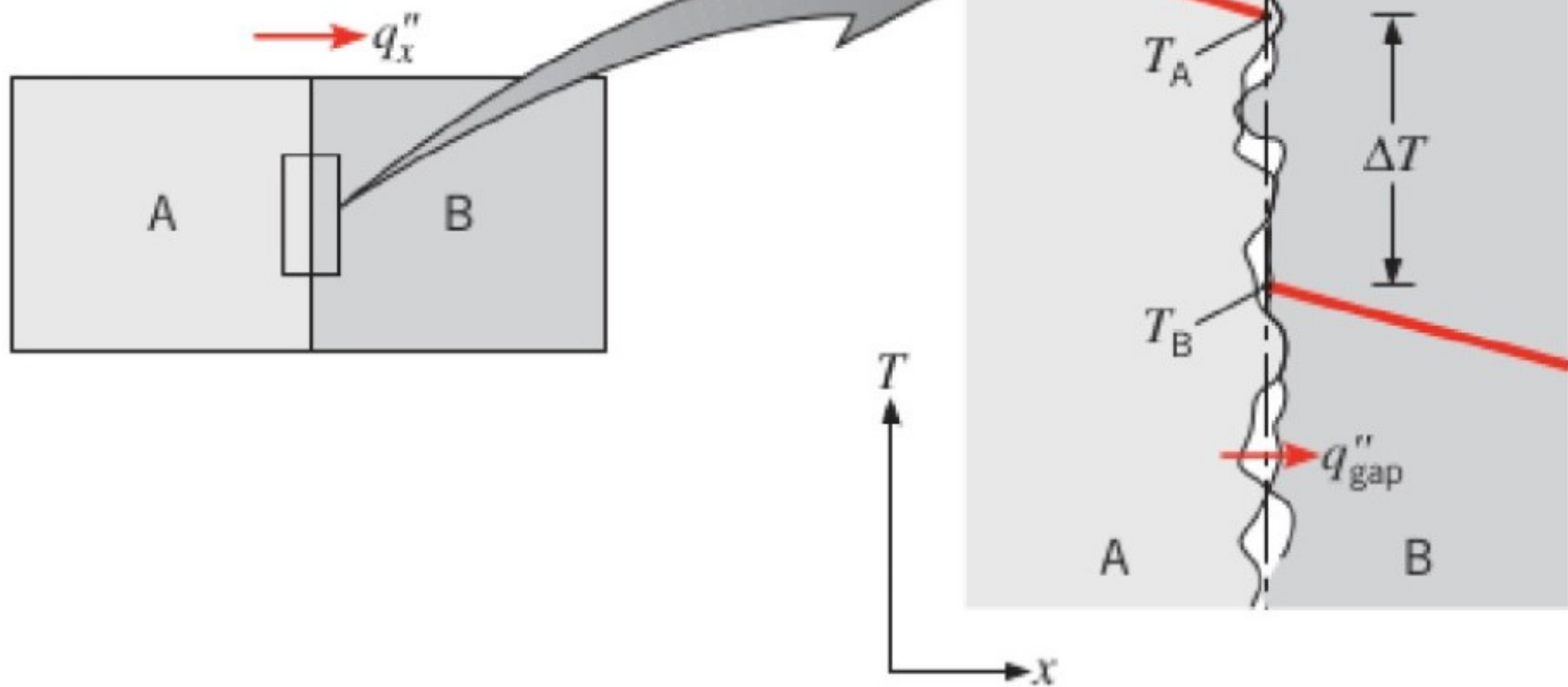


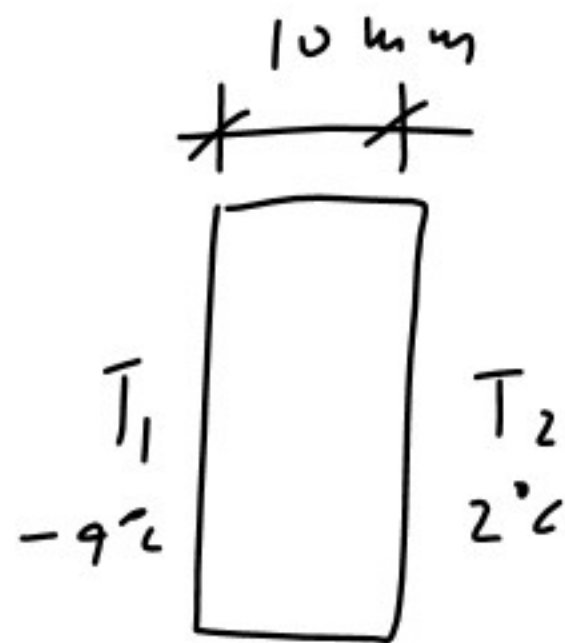
TABLE 3.1 Thermal contact resistance for (a) metallic interfaces under vacuum conditions and (b) aluminum interface (10- μm surface roughness, 10^5 N/m^2) with different interfacial fluids [1]

Thermal Resistance, $R''_{t,c} \times 10^4 \text{ (m}^2 \cdot \text{K/W)}$

(a) Vacuum Interface			(b) Interfacial Fluid	
Contact pressure	100 kN/m^2	10,000 kN/m^2	Air	2.75
Stainless steel	6–25	0.7–4.0	Helium	1.05
Copper	1–10	0.1–0.5	Hydrogen	0.720
Magnesium	1.5–3.5	0.2–0.4	Silicone oil	0.525
Aluminum	1.5–5.0	0.2–0.4	Glycerine	0.265

3.7 A $t = 10$ -mm-thick horizontal layer of water has a top surface temperature of $T_c = -4^\circ\text{C}$ and a bottom surface temperature of $T_h = 2^\circ\text{C}$. Determine the location of the solid-liquid interface at steady state.

Assume k constant



$$T(x) = (T_2 - T_1) \frac{x}{L} + T_1$$

$$= (2 - -4) \frac{x}{10 \text{ mm}} - 4 = \frac{6^\circ\text{C}}{10 \text{ mm}} x - 4^\circ\text{C}$$

$$T(x) = 0^\circ\text{C} = \frac{6^\circ\text{C}}{10 \text{ mm}} x - 4^\circ\text{C} \Rightarrow 4^\circ\text{C} = \frac{6^\circ\text{C}}{10 \text{ mm}} x$$

$$\frac{10 \text{ mm} \cdot 4^\circ\text{C}}{6^\circ\text{C}} = x = \boxed{6.67 \text{ mm}}$$