1.5

The concrete slab of a basement is 11 m long, 8 m wide, and 0.20 m thick. During the winter, temperatures are nominally 17°C and 10°C at the top and bottom surfaces, respectively. If the concrete has a thermal conductivity of 1.4 W/m·K, what is the rate of heat loss through the slab? If the basement is heated by a gas furnace operating at an efficiency of $\eta_f = 0.90$ and natural gas is priced at $C_g = \$0.02/\text{MJ}$, what is the daily cost of the heat loss?

$$q'' = -k\,\frac{T_2 - T_1}{L}$$

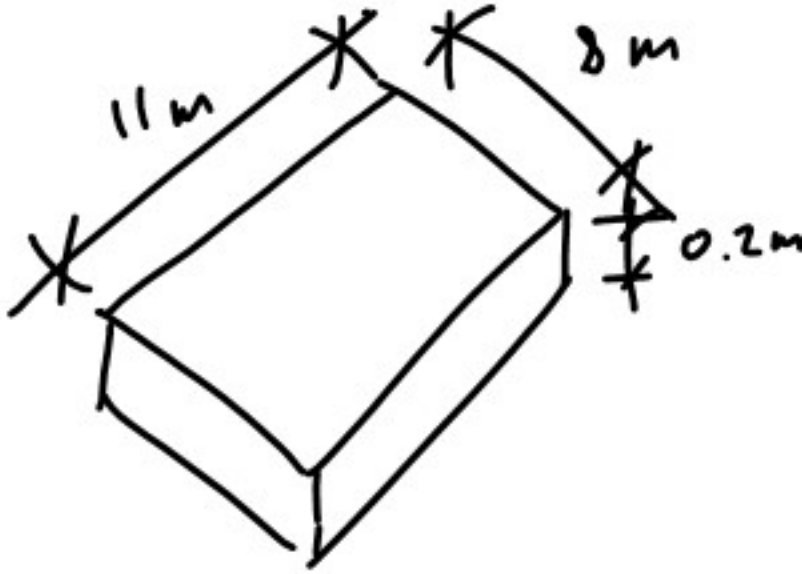
$$= -1.4\,\frac{\text{W}}{\text{mK}}\;\frac{10°\text{C} - 17°\text{C}}{0.2\text{ m}}$$

$$= -1.4\,\frac{\text{W}}{\text{mK}}\;\frac{-7°\text{C}}{0.2\text{ m}}$$

$$= -1.4\,\frac{\text{W}}{\text{m}\cancel{\text{K}}}\;\frac{-7\cancel{\text{K}}}{0.2\text{m}}$$

$$= 49\,\frac{\text{W}}{\text{m}^2}$$

$$q = 49\,\frac{\text{W}}{\cancel{\text{m}^2}}\;11\cancel{\text{m}}\;8\cancel{\text{m}} = \boxed{4312\text{ W}}$$



A = 11m 8m

$$W_f \eta_f = q$$

$$W_f = \frac{q}{\eta_f} = \frac{4312}{0.9} = 4791 \text{ W}$$

$$W_f \ 1\,\text{day} = 4791 \text{ W } 1\,\text{day} \ \frac{24 \text{ h}}{1\,\text{day}} \ \frac{60 \text{ m}}{1 \text{ h}} \ \frac{60 \text{ s}}{1 \text{ m}} = 4.14 \times 10^8 \text{ Ws}$$

$$= 4.14 \times 10^8 \text{ J}$$

$$= 414 \text{ MJ}$$

$$414 \cancel{\text{MJ}} \ \frac{\$0.02}{\cancel{\text{MJ}}} = \boxed{\$8.28}$$

1.6

The heat flux through a wood slab 50 mm thick, whose inner and outer surface temperatures are 40 and 20°C, respectively, has been determined to be 40 W/m^2. What is the thermal conductivity of the wood?

$$q'' = -k \frac{T_2 - T_1}{L}$$

$$40 \frac{W}{m^2} = -k \frac{20°C - 40°C}{50\,mm}$$

$$-\frac{40 \frac{W}{m^2}\ 50\,mm}{20°C - 40°C} = k = 100 \frac{W\ \cancel{mm}}{m^2\ °C} \frac{1\ \cancel{m}}{1000\ \cancel{mm}} = 0.1 \frac{W}{m\,°C} = \boxed{0.1 \frac{W}{m\,K}}$$

1.10

A wall is made from an inhomogeneous (nonuniform) material for which the thermal conductivity varies through the thickness according to $k = ax + b$, where a and b are constants. The heat flux is known to be constant. Determine expressions for the temperature gradient and the temperature distribution when the surface at $x = 0$ is at temperature T_1.

$$q'' = -k \frac{dT}{dx}$$

$$q'' = -(ax+b) \frac{dT}{dx}$$

$$\int \frac{-q''}{ax+b}\, dx = \int dT$$

$$-q'' \frac{\log(ax+b)}{a} + C = T$$

1.12

You've experienced convection cooling if you've ever extended your hand out the window of a moving vehicle or into a flowing water stream. With the surface of your hand at a temperature of 30°C, determine the convection heat flux for (a) a vehicle speed of 40 km/h in air at –8°C with a convection coefficient of 40 $W/m^2 \cdot K$ and (b) a velocity of 0.2 m/s in a water stream at 10°C with a convection coefficient of 900 $W/m^2 \cdot K$. Which condition would *feel* colder? Contrast these results with a heat flux of approximately 30 W/m^2 under normal room conditions.

$$q'' = h(T_s - T_\infty)$$

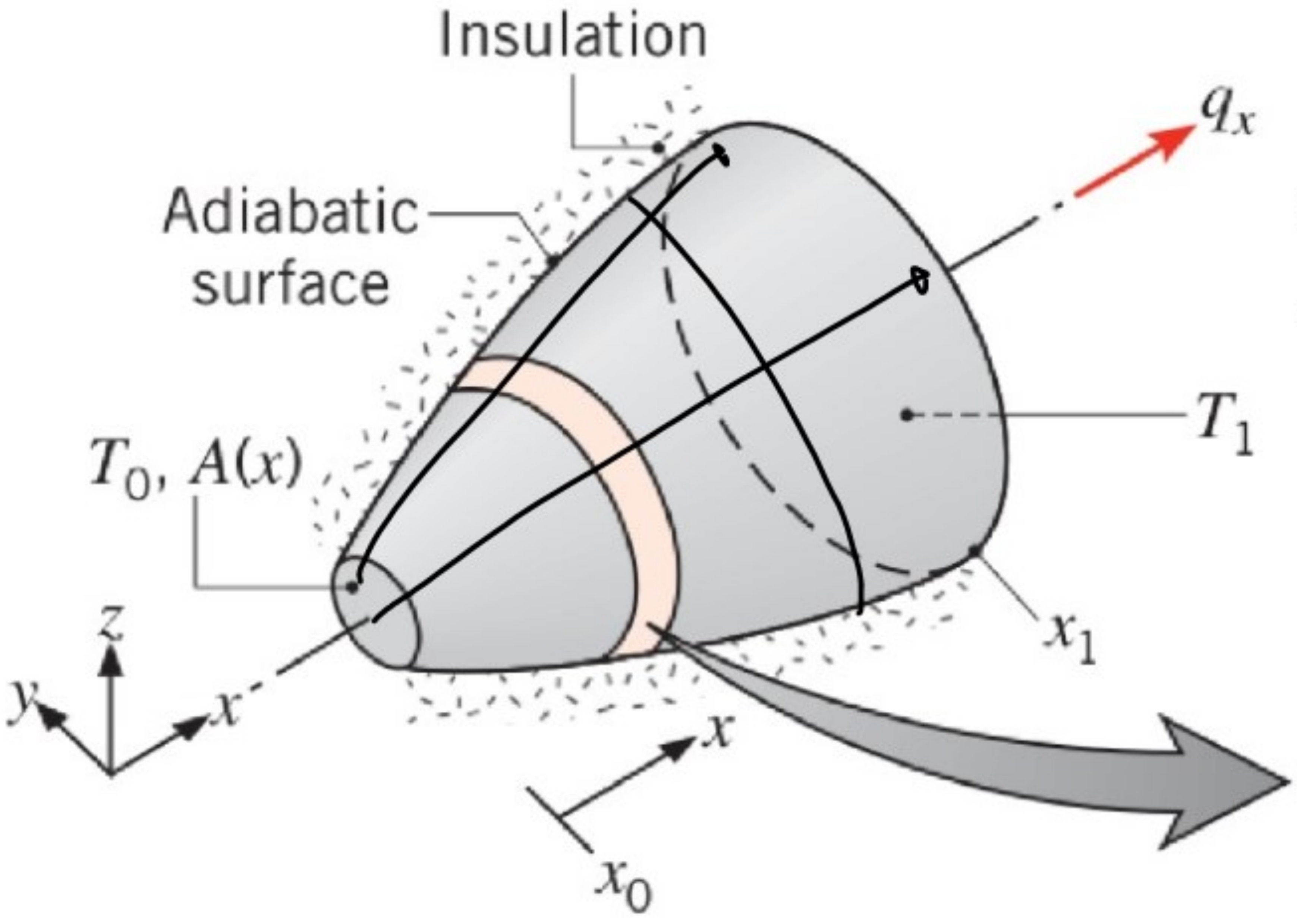
a. $q'' = 40 \frac{W}{m^2 K}(30°C + 8°C)$

$= 1520\ W/m^2$

b. $q'' = 900 \frac{W}{m^2 K}(30°C - 10°C)$

$= 18000 \frac{W}{m^2}$

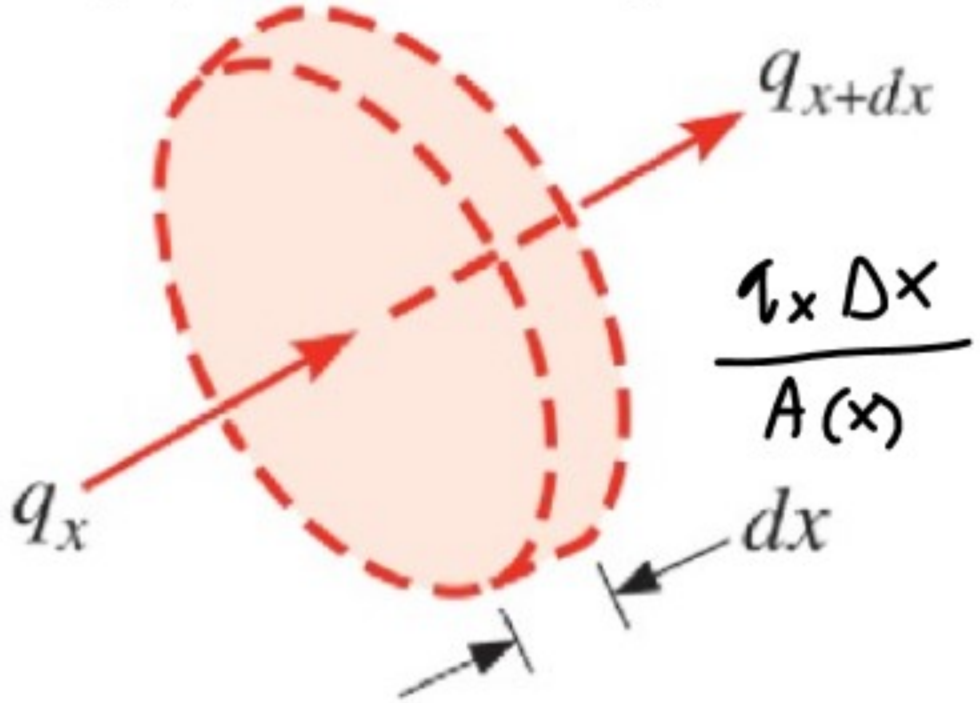
1.5 D Conduction



$$q_x = -k A(x) \frac{\partial T}{\partial x}$$

$$\frac{q_x}{A(x)} \partial x = -k \, \partial T$$

$$q_x \int \frac{1}{A(x)} dx = -k \int dT$$



$$\frac{q_x \Delta x}{A(x)} = -k \Delta T$$

$$q_x \int \frac{1}{A(x)}\, dx = -k \int_{T_1}^{T} dT$$

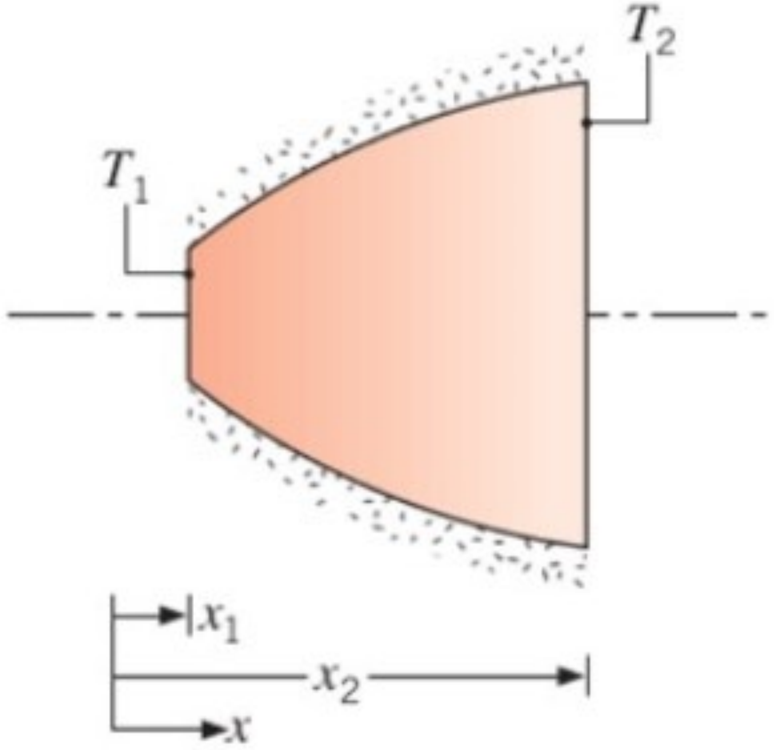
The diagram shows a conical section fabricated from pure aluminum. It is of circular cross section having diameter $D = ax^{1/2}$, where $a = 0.5\ \mathrm{m}^{1/2}$. The small end is located at $x_1 = 25$ mm and the large end at $x_2 = 125$ mm. The end temperatures are $T_1 = 600$ K and $T_2 = 400$ K, while the lateral surface is well insulated.

$$A(x) = \pi r^2 = \pi \frac{D^2}{4}$$

$$= \pi \frac{(a x^{1/2})^2}{4}$$

$$= \frac{\pi}{4} a^2 x$$

$$q_x \int_{x_1}^{x} \frac{4}{\pi a^2 x}\, dx = -k(T - T_1)$$



$$\boxed{\frac{4 q_x}{\pi a^2} \ln(x/x_1) = -k\,(T - T_1)}$$

$$\frac{4 q_x}{\pi a^2} \ln(x) \Big|_{x_1}^{x} = -k(T - T_1)$$

$$\frac{4 q_x}{\pi a^2} \left(\ln(x) - \ln(x_1)\right) = -k(T - T_1)$$

(a) Derive an expression for the temperature distribution $T(x)$ in symbolic form, assuming one-dimensional conditions. Sketch the temperature distribution.

(b) Calculate the heat rate q_x.

$$\frac{4 q_x}{\pi a^2} \ln\left(x / x_1\right) = -k\left(T - T_1\right)$$

$$q_x = \frac{-k\left(T - T_1\right) \pi a^2}{4 \ln\left(x / x_1\right)}$$

$$= \frac{-k\left(400\,K - 600\,K\right) \pi \left(0.5\,m^{1/2}\right)^2}{4 \ln\left(125\,mm / 25\,mm\right)} =$$