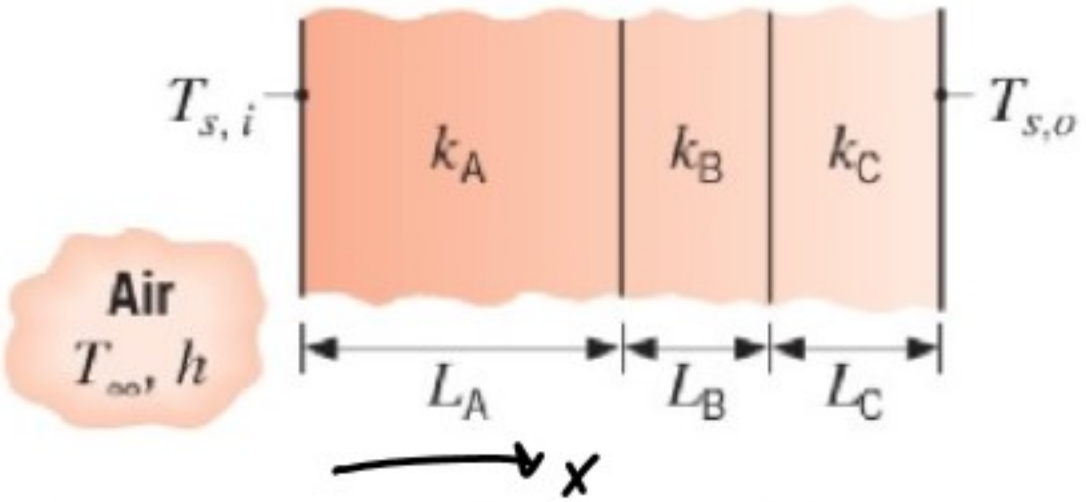
The composite wall of an oven consists of three materials, two of which are of known thermal conductivity, $k_A = 25$ W/m · K and $k_C = 60$ W/m · K, and known thickness, $L_A = 0.40$ m and $L_C = 0.20$ m. The third material, B, which is sandwiched between materials A and C, is of known thickness, $L_B = 0.20$ m, but unknown thermal conductivity k_B.



Under steady-state operating conditions, measurements reveal an outer surface temperature of $T_{s,o} = 20°C$, an inner surface temperature of $T_{s,i} = 600°C$, and an oven air temperature of $T_\infty = 800°C$. The inside convection coefficient h is known to be 25 W/m² · K. What is the value of k_B?

$$q'' = h(T_{si} - T_\infty)$$

$$q'' = 25 \frac{W}{m^2 K}(600°C - 800°C)$$

$$q'' = 25 \frac{W}{m^2 \cancel{K}}(-200 \cancel{K})$$

$$= -5000 \frac{W}{m^2}$$

$$R_A = \frac{L_A}{K_A A}$$

$$= \frac{0.4 \text{ m}}{25 \frac{W}{mK} A}$$

$$= \frac{0.016}{A} \frac{m^2 K}{W}$$

$$R_B = \frac{L_B}{K_B A} = \frac{0.2\,m}{K_B A}$$

$$R_C = \frac{L_C}{K_C A} = \frac{0.2\,m}{60\,\frac{W}{mK}\,A} = \frac{0.033}{A}\,\frac{m^2 K}{W}$$

$$R_e = R_A + R_B + R_C = \frac{0.016}{A}\,\frac{m^2 K}{W} + \frac{0.2\,m}{K_B A} + \frac{0.033}{A}\,\frac{m^2 K}{W} = \frac{0.049}{A}\,\frac{m^2 K}{W} + \frac{0.2\,m}{K_B A}$$

$$\frac{T_{so} - T_{si}}{q} = \frac{0.049}{A}\,\frac{m^2 K}{W} + \frac{0.2\,m}{K_B A}$$

$$\frac{T_{so} - T_{si}}{q''} = 0.049\,\frac{m^2 K}{W} + \frac{0.2\,m}{K_B}$$

$$\frac{20^\circ C - 100^\circ C}{-5000 \frac{W}{m^2}} = 0.049 \frac{m^2 K}{W} + \frac{0.2 \ m}{K_B}$$

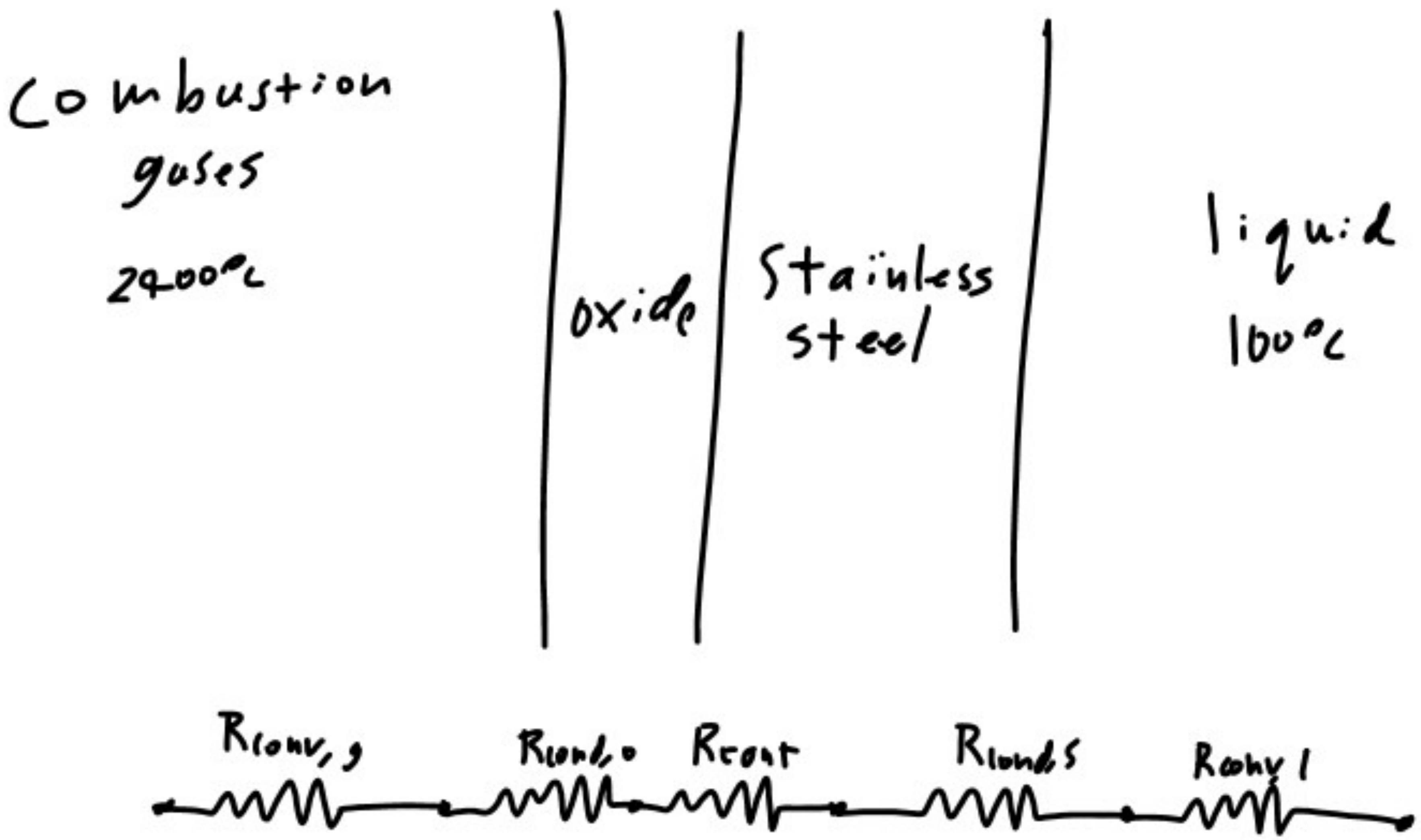
$$\frac{-580 \ K}{-5000 \frac{W}{m^2}} = 0.049 \frac{m^2 K}{W} + \frac{0.2 \ m}{K_B}$$

$$0.116 \frac{K m^2}{W} - 0.049 \frac{m^2 K}{W} = \frac{0.2 \ m}{k_B}$$

$$0.067 \frac{K m^2}{W} = \frac{0.2 \ m}{k_B}$$

$$K_B = \frac{0.2 \ m}{0.067 \frac{K m^2}{W}} = \boxed{3 \frac{W}{mK}}$$

A composite wall separates combustion gases at 2400°C from a liquid coolant at 100°C, with gas and liquid-side convection coefficients of 25 and 1000 $W/m^2 \cdot K$. The wall is composed of a 12-mm-thick layer of beryllium oxide on the gas side and a 24-mm-thick slab of stainless steel (AISI 304) on the liquid side. The contact resistance between the oxide and the steel is 0.05 $m^2 \cdot K/W$. What is the rate of heat loss per unit surface area of the composite? Sketch the temperature distribution from the gas to the liquid.



$$R_t = R_{conv,g} + R_{cond,o} + R_{cont} + R_{cond,s} + R_{conv,l}$$

$$R_{conv,g} = \frac{1}{hA} = \frac{1}{25\frac{W}{m^2K}A}$$

$$R_{conv,l} = \frac{1}{hA} = \frac{1}{1000\frac{W}{m^2K}A}$$

$$R_{cont} = \frac{0.05\ \frac{m^2K}{W}}{A}$$

$$R_{cond,o} = \frac{L}{KA} = \frac{0.012\,m}{272\ \frac{W}{mK}A}$$

$$R_{cond,s} = \frac{L}{KA} = \frac{0,024}{19.9\frac{W}{mK}A}$$

$$R_c = \frac{1}{25\,\frac{W}{m^2K}\,A} + \frac{1}{1000\,\frac{W}{m^2K}\,A} + \frac{0.05\,\frac{m^2K}{W}}{A} + \frac{0.012\ m}{272\,\frac{W}{mK}\,A} + \frac{0.024\ m}{19.9\,\frac{W}{mK}\,A}$$