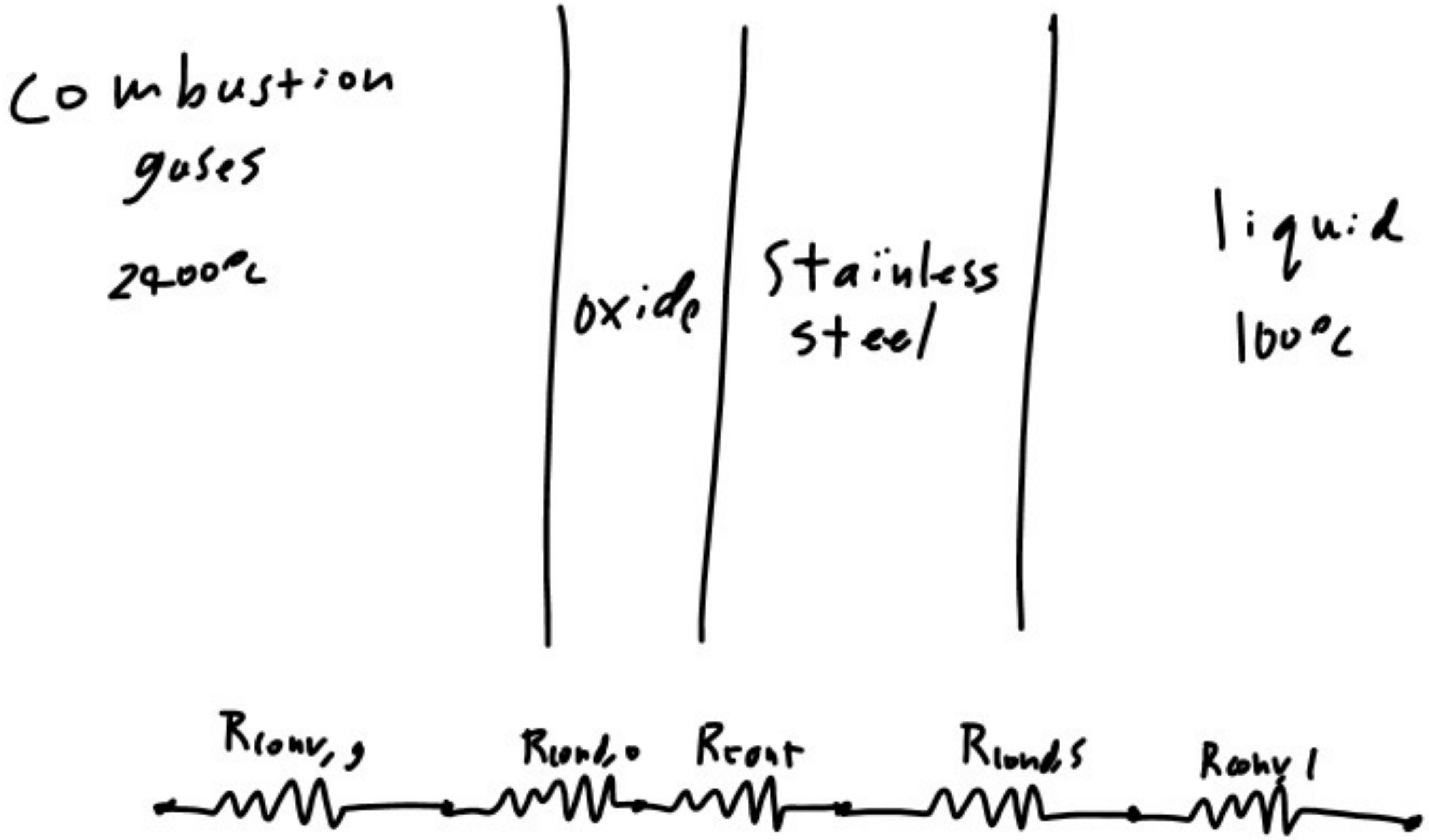
A composite wall separates combustion gases at 2400°C from a liquid coolant at 100°C, with gas and liquid-side convection coefficients of 25 and 1000 $W/m^2 \cdot K$. The wall is composed of a 12-mm-thick layer of beryllium oxide on the gas side and a 24-mm-thick slab of stainless steel (AISI 304) on the liquid side. The contact resistance between the oxide and the steel is 0.05 $m^2 \cdot K/W$. What is the rate of heat loss per unit surface area of the composite? Sketch the temperature distribution from the gas to the liquid.



$$R_t = R_{conv,g} + R_{cond,o} + R_{cont} + R_{cond,s} + R_{conv,l}$$

$$R_{conv,g} = \frac{1}{hA} = \frac{1}{25\frac{W}{m^2K}A}$$

$$R_{conv,l} = \frac{1}{hA} = \frac{1}{1000\frac{W}{m^2K}A}$$

$$R_{cont} = \frac{0.05\ \frac{m^2K}{W}}{A}$$

$$R_{cond,o} = \frac{L}{KA} = \frac{0.012\ m}{272\ \frac{W}{mK}A}$$

$$R_{cond,s} = \frac{L}{KA} = \frac{0.024}{14.9\frac{W}{mK}A}$$

$$R_e = \frac{1}{25 \frac{W}{m^2 K} A} + \frac{1}{1000 \frac{W}{m^2 K} A} + \frac{0.05 \frac{m^2 K}{W}}{A} + \frac{0.012\ m}{272 \frac{W}{mK} A} + \frac{0.024\ m}{19.9 \frac{W}{mK} A}$$

$$= \frac{0.0927 \frac{m^2 K}{W}}{A}$$

$$R_e = \frac{T_2 - T_1}{q} \qquad \frac{0.0927 \frac{m^2 K}{W}}{A} = \frac{100°C - 2400°C}{q}$$

$$\frac{q}{A} = q'' = \frac{-2300\ K}{0.0927 \frac{m^2 K}{W}} = \boxed{-24.8 \frac{kW}{m^2}}$$

1D conduction with generation

$$\nabla^2 T + \frac{\dot{q}}{k} = \frac{1}{\alpha}\cancel{\frac{\partial T}{\partial t}}^{\,0}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = 0$$

$$T = \frac{-\dot{q}}{2k}x^2 + C_1 x + C_2$$

$$\boxed{T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s2} - T_{s1}}{2}\frac{x}{L} + \frac{T_{s1} + T_{s2}}{2}}$$

$$\underline{V} = I R_e$$

$$\dot{E}_g = I^2 R_e = I\underline{V} = \frac{\underline{V}^2}{R_e}$$

$$\dot{q} \equiv \frac{\dot{E}_g}{V}$$

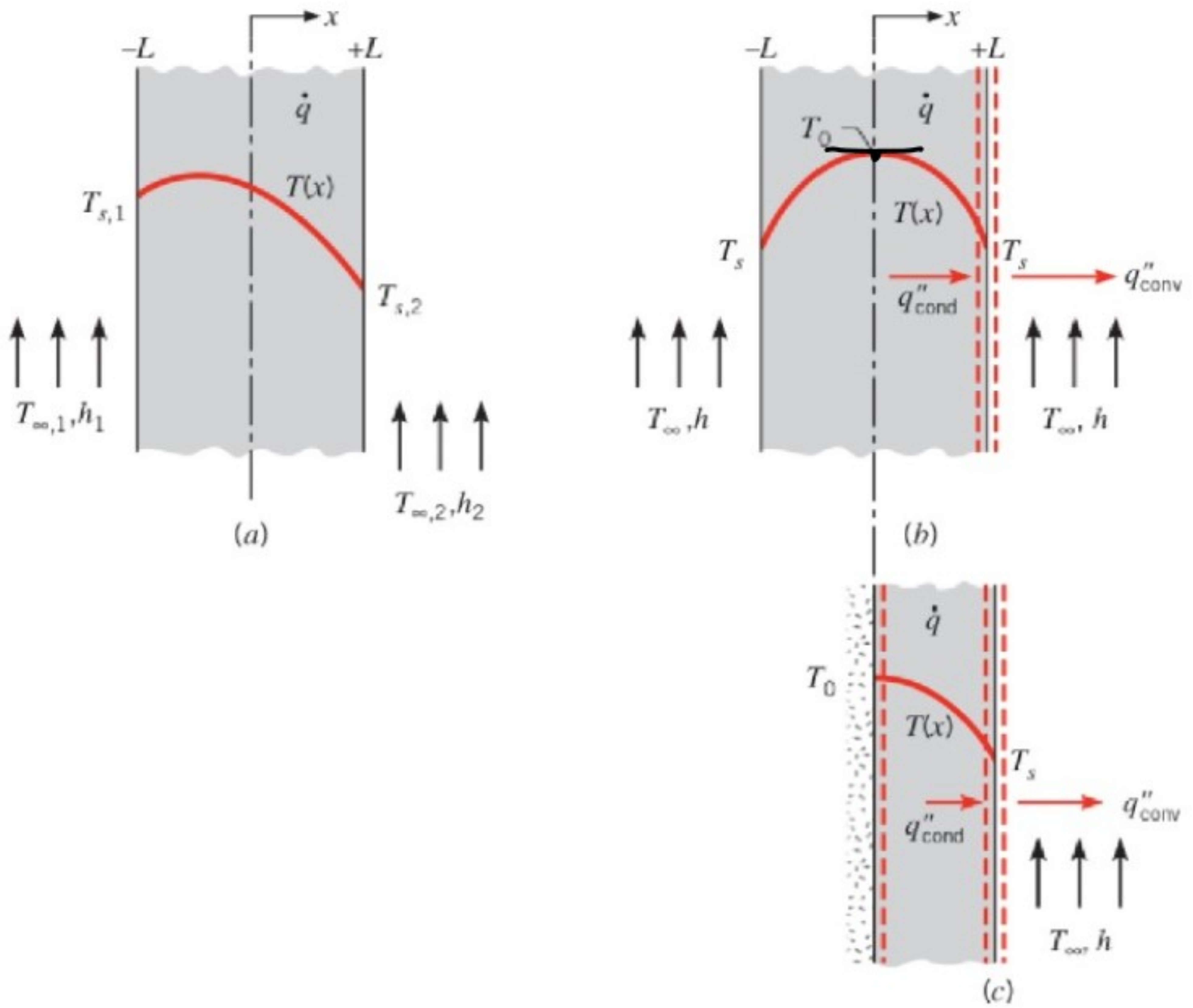
V volume

$\underline{V}$ voltage

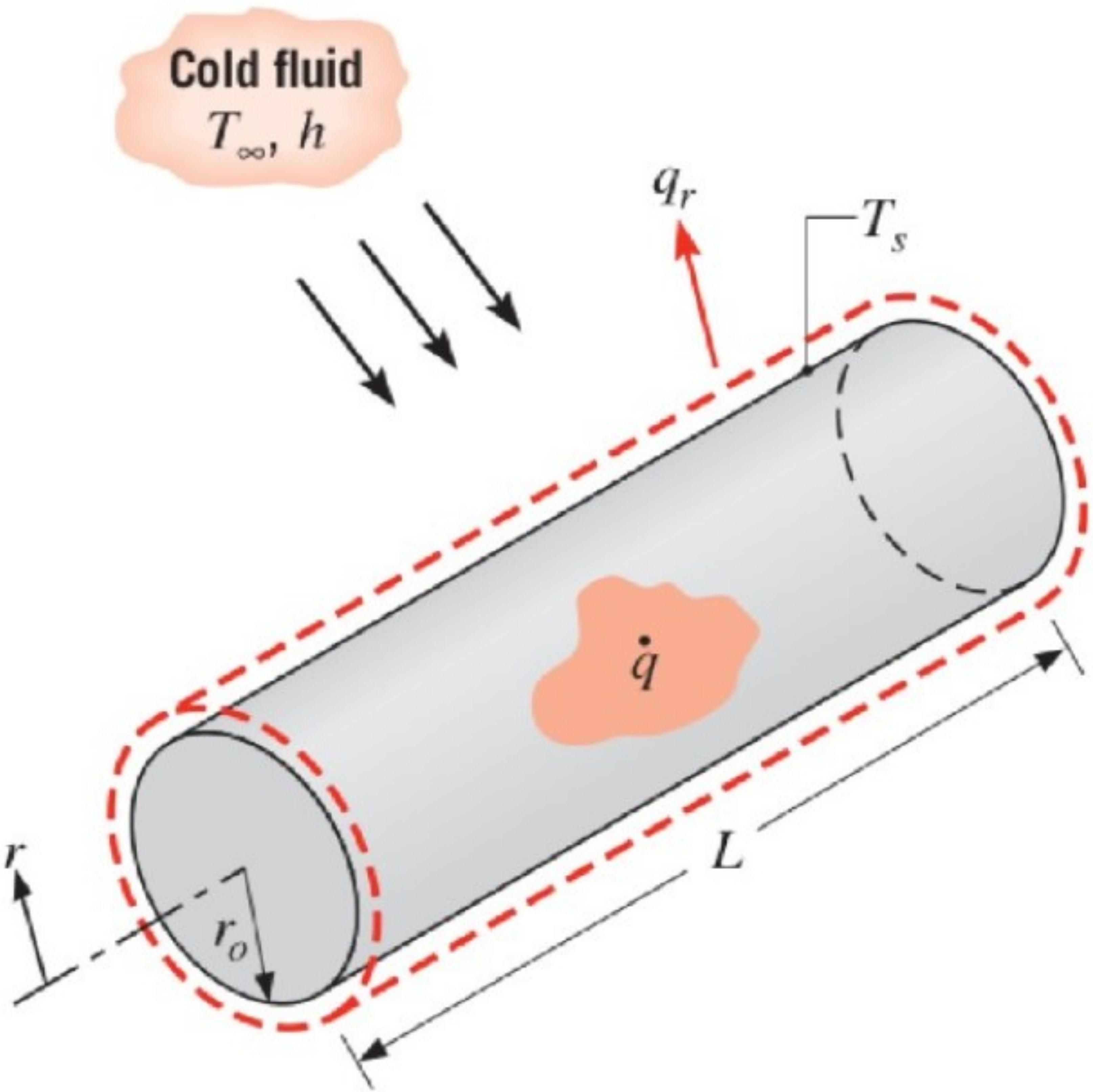
if $T_{s1} = T_{s2} = T_s$

$$\boxed{T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_s}$$

Maximum $\boxed{T(0) = \frac{\dot{q}L^2}{2k} + T_s}$



x
−L
+L
q̇
T(x)
$T_{s,1}$
$T_{s,2}$
$T_{\infty,1}, h_1$
$T_{\infty,2}, h_2$
(a)
T_0
T_s
q''_{cond}
q''_{conv}
T_∞, h
(b)
(c)


Cold fluid
T_∞, h
q_r
T_s
$\dot{q}$
r
r_o
L

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\underbrace{\cancel{\frac{\partial^2 T}{\partial \phi^2}}}_{0}+\underbrace{\cancel{\frac{\partial^2 T}{\partial z^2}}}_{0}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\underbrace{\cancel{\frac{\partial T}{\partial t}}}_{0}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\dot{q}}{k}=0$$

$$T(r)=\frac{-\dot{q}}{4k}r^2+C_1\ln r+C_2$$

$$\left.\frac{dT}{dr}\right|_{r=0}=0 \qquad T(r_0)=T_s$$

$$\boxed{T(r)=\frac{\dot{q}r_0^2}{4k}\left(1-\frac{r^2}{r_0^2}\right)+T_s}$$

$$\dot{q}\,\pi r_0^2 L = h\,2\pi r_0 L(T_s-T_\infty)$$

$$\boxed{T_s=T_\infty+\frac{\dot{q}r_0}{2h}}$$

An uncoated, solid cable of length $L = 1$ m and diameter $D = 40$ mm is exposed to convection conditions characterized by $h = 55\ \text{W/m}^2 \cdot \text{K}$ and $T_\infty = 20°\text{C}$. Determine the maximum electric current that can be carried by the cable if it is pure copper, pure aluminum, or pure tin. Calculate the corresponding minimum wire temperatures. The electrical resistivity is $\rho_e = 10 \times 10^{-8}$ $\Omega \cdot \text{m}$ for copper and aluminum at their melting points, while the electrical resistivity of tin is $\rho_e = 20 \times 10^{-8}$ $\Omega \cdot \text{m}$ at its melting point.

r = 20mm

$$\dot{E}_g = I^2 R_e = I^2 \frac{\rho_e}{L}$$

$$\dot{q} = \frac{\dot{E}_g}{V} = \frac{I^2 \frac{\rho_e}{L}}{\pi r_0^2 L} = \frac{I^2 \rho_e}{\pi r_0^2 L^2}$$

$$T_s = T_\infty + \frac{\dot{q} r_0}{2h}$$

$$T(r) = \frac{\dot{q} r_0^2}{4K}\left(1 - \frac{r^2}{r_0^2}\right) + T_s$$

$$T(0) = \frac{\dot{q} r_0^2}{4K} + T_s$$

$$= \frac{\dot{q} r_0^2}{4K} + T_\infty + \frac{\dot{q} r_0}{2h}$$

$$T(0) - T_\infty = \dot{q}\left(\frac{r_0^2}{4K} + \frac{r_0}{2h}\right)$$

$$\dot{q} = \frac{T(0) - T_\infty}{\frac{r_0^2}{4K} + \frac{r_0}{2h}}$$

$$\dot{q} = \frac{1358\,K - 20\,°C}{\dfrac{(0.02\,m)^2}{4\left(401\,\frac{W}{m\,K}\right)} + \dfrac{0.02\,m}{2\left(55\,\frac{W}{m^2 K}\right)}} =$$