An uncoated, solid cable of length $L = 1$ m and diameter $D = 40$ mm is exposed to convection conditions characterized by $h = 55 \text{ W/m}^2 \cdot \text{K}$ and $T_\infty = 20°\text{C}$. Determine the maximum electric current that can be carried by the cable if it is pure copper, pure aluminum, or pure tin. Calculate the corresponding minimum wire temperatures. The electrical resistivity is $\rho_e = 10 \times 10^{-8}$ $\Omega \cdot \text{m}$ for copper and aluminum at their melting points, while the electrical resistivity of tin is $\rho_e = 20 \times 10^{-8}$ $\Omega \cdot \text{m}$ at its melting point.

r = 20mm

$$\dot{E}_g = I^2 R_e = I^2 \frac{\rho_e}{L}$$

$$\dot{q} = \frac{\dot{E}_g}{V} = \frac{I^2 \frac{\rho_e}{L}}{\pi r_0^2 L} = \frac{I^2 \rho_e}{\pi r_0^2 L^2}$$

$$T_s = T_\infty + \frac{\dot{q} r_0}{2h}$$

$$T(r) = \frac{\dot{q} r_0^2}{4K}\left(1 - \frac{r^2}{r_0^2}\right) + T_s$$

$$T(0) = \frac{\dot{q} r_0^2}{4K} + T_s$$

$$= \frac{\dot{q} r_0^2}{4K} + T_\infty + \frac{\dot{q} r_0}{2h}$$

$$T(0) - T_\infty = \dot{q}\left(\frac{r_0^2}{4K} + \frac{r_0}{2h}\right)$$

$$\dot{q} = \frac{T(0) - T_\infty}{\frac{r_0^2}{4K} + \frac{r_0}{2h}}$$

$$\dot{q} = \frac{1358\,K - 20^\circ C}{\frac{(0.02\,m)^2}{4\left(401\,\frac{W}{mK}\right)} + \frac{0.02\,m}{2\left(55\,\frac{W}{m^2K}\right)}} = \frac{1358\,K - 293.15\,K}{2.49\times10^{-7}\,\frac{m^3K}{W} + 1.8\times10^{-4}\,\frac{m^3K}{W}}$$

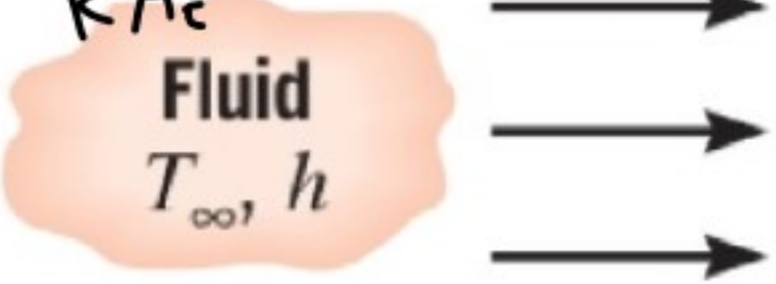
$$= \frac{1065\,K}{1.8\times10^{-4}\,\frac{m^3K}{W}} = 5.92\times10^{6}\,\frac{W}{m^3}$$

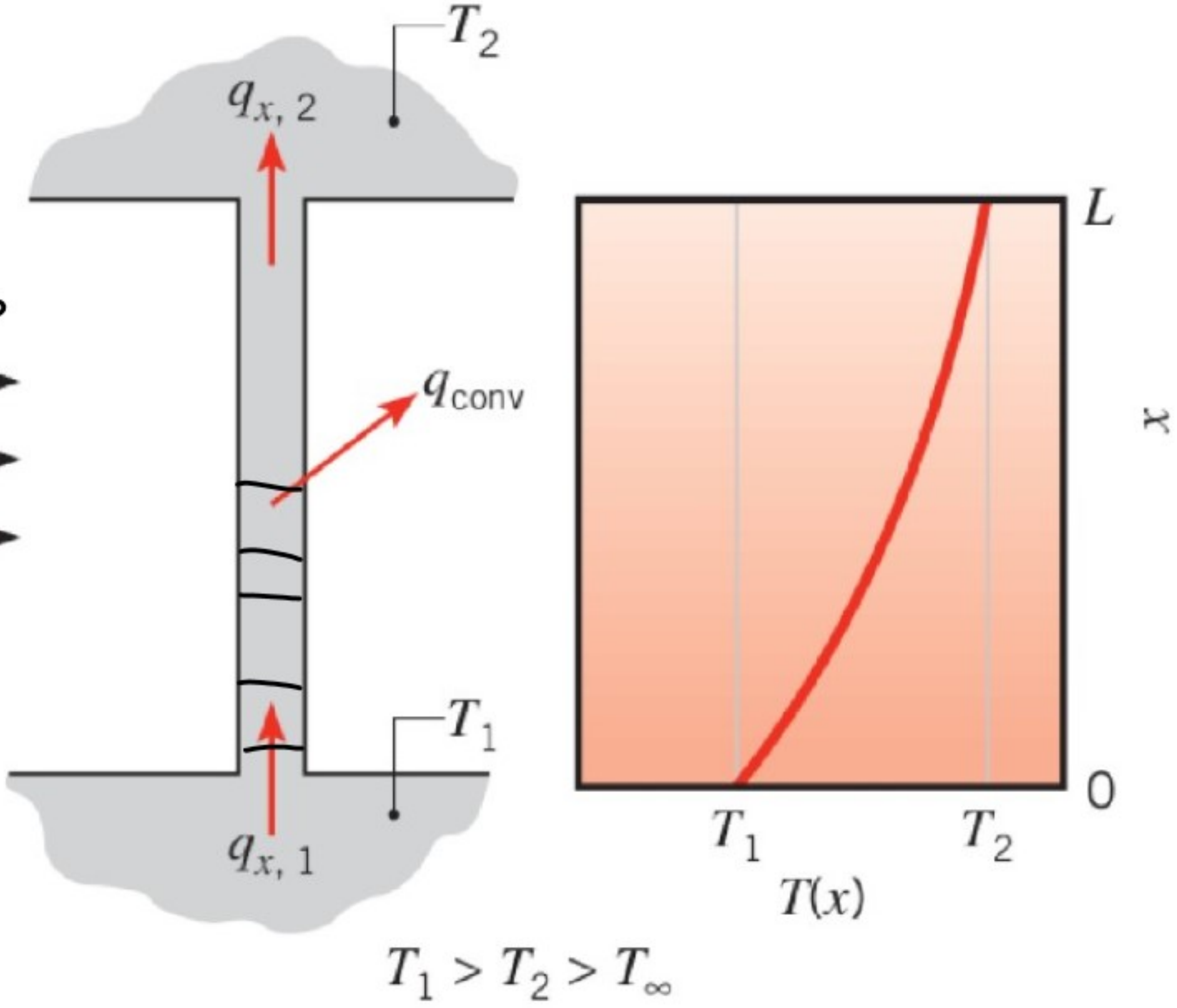
$$I = \sqrt{\frac{\dot{q}\,\pi r_o^2 L^2}{\rho_e}} = \sqrt{\frac{5.92\times10^{6}\,\frac{W}{\cancel{m^3}}\,\pi\,(0.02\,\cancel{m})^2\,(1\,\cancel{m})^2}{10\times10^{-8}\,\Omega\,\cancel{m}}} = 2.73\times10^{5}\sqrt{\frac{W}{\Omega}} = \boxed{2.73\times10^{5}\ A}$$

Extended Surfaces

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

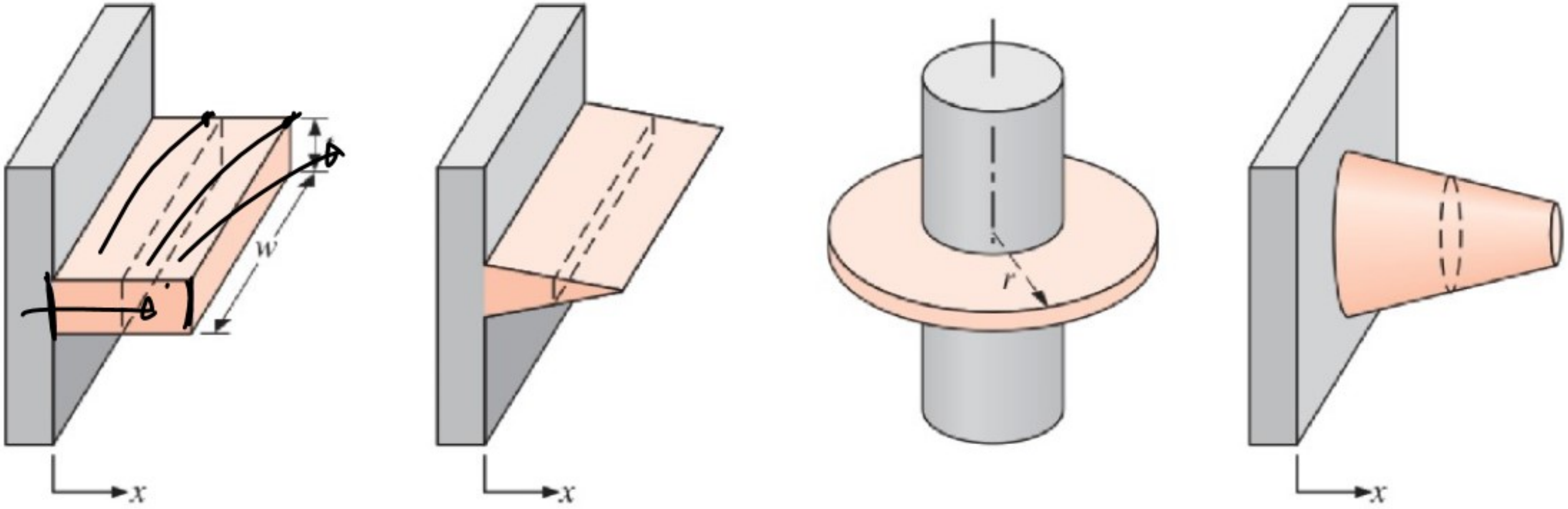
$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T(x) - T_\infty) = 0$$





A_c cross sectional area

P perimiter



w
x
x
r
x

$$\theta(x) \equiv T(x) - T_\infty$$

$$\frac{d^2\theta}{dx^2} - m^2 \theta = 0 \qquad m^2 \equiv \frac{hP}{kA_c}$$

$$\theta(x) = c_1 e^{mx} + c_2 e^{-mx}$$

$$\theta(0) = T_b - T_\infty \equiv \theta_b$$

TABLE 3.4 Temperature distributions and heat rates for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection: $h\theta(L) = -k\,d\theta/dx\vert_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$ (3.75)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$ (3.77)
B	Adiabatic: $d\theta/dx\vert_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$ (3.80)	$M \tanh mL$ (3.81)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$ (3.82)	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ (3.83)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx} (3.84)	M (3.85)

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$

A table of hyperbolic functions is given in Appendix B.1.

Fin performance

Fin ~~effectiveness~~

$$\varepsilon_f = \frac{q_f}{h A_{cb} \theta_b}$$

A_{cb} fin cross sectional area at base

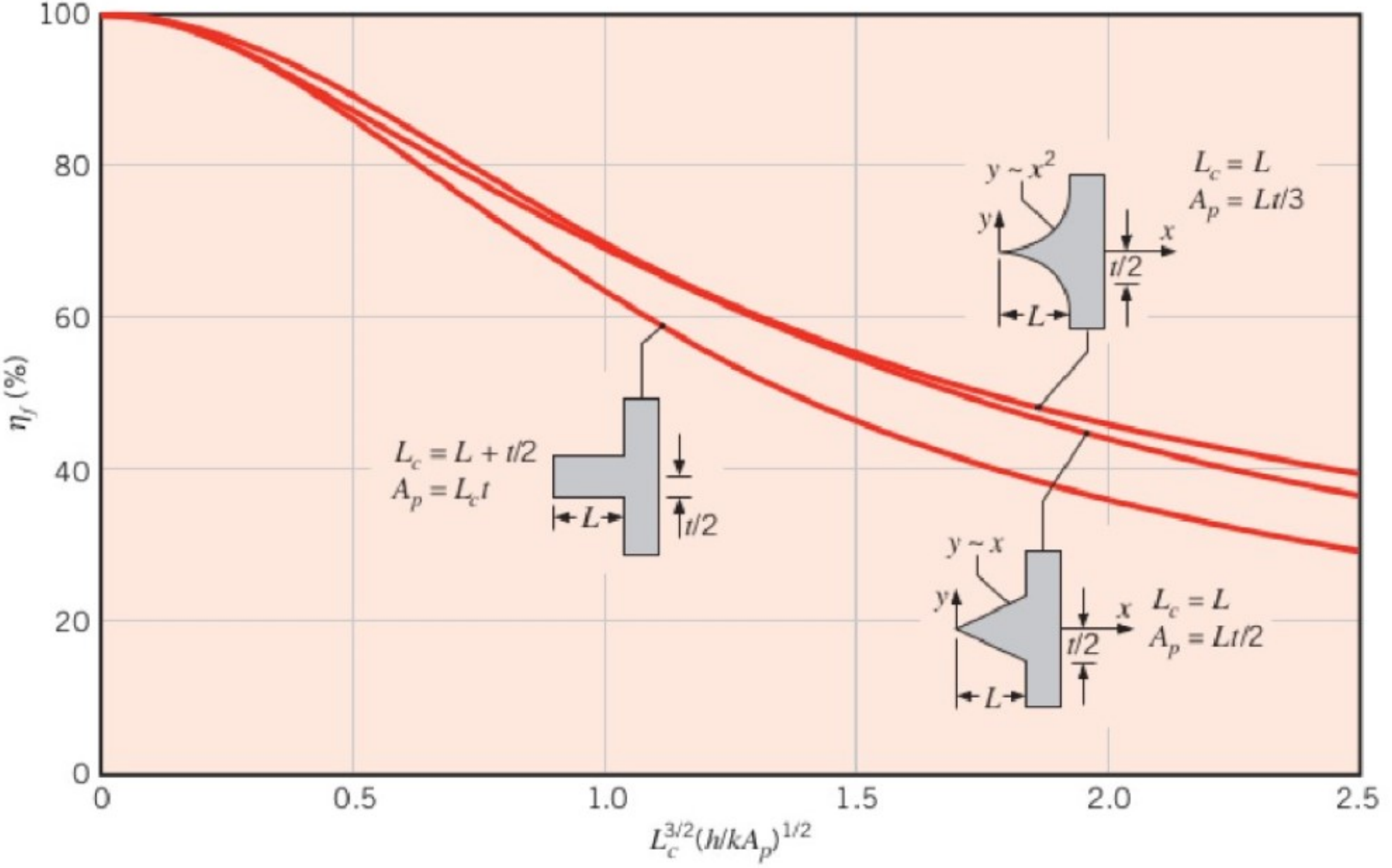
infinitly long fin w/ const A_c

$$\varepsilon_f = \left(\frac{kP}{hA_c}\right)^{1/2}$$

$$R_f = \frac{\theta_b}{q_f}$$

Fin efficiency

$$\eta_f \equiv \frac{q_f}{q_{max}} = \frac{q_f}{hA_f\,(T_b - T_\infty)} = \frac{q_f}{hA_f\,\theta_b}$$



100
80
60
40
20
0
0
0.5
1.0
1.5
2.0
2.5
η_f (%)
$L_c^{3/2}(h/kA_p)^{1/2}$
$y \sim x^2$
$L_c = L$
$A_p = Lt/3$
$L_c = L + t/2$
$A_p = L_c t$
$y \sim x$
$L_c = L$
$A_p = Lt/2$
$t/2$
L