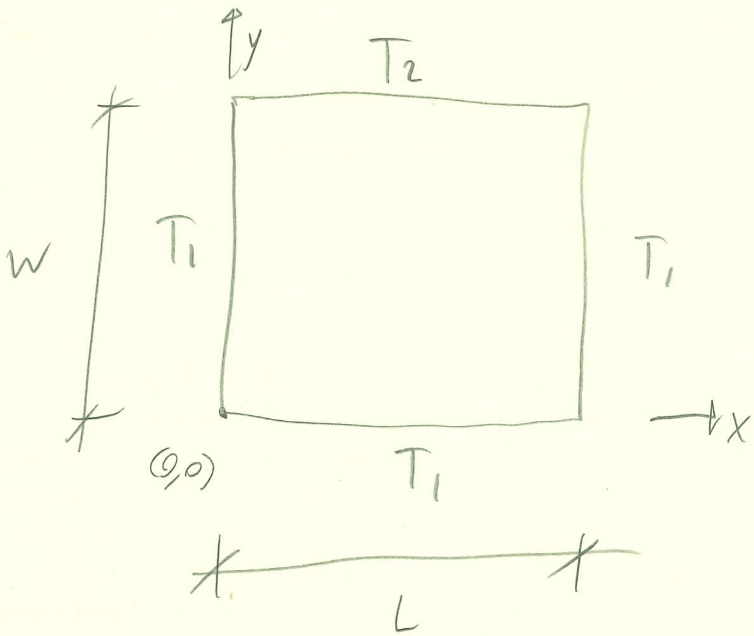
2D conduction

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = 0$$



$\theta(0,y)=0$

$\theta(L,y)=0$

$\theta(x,0)=0$

$\theta(x,W)=1$

$$\boxed{\theta(x,y) = \frac{T(x,y) - T_1}{T_2 - T_1}}$$

if $T(x,y) = T_1 \qquad \theta = 0$

$T(x,y) = T_2 \qquad \theta = 1$

$$\frac{\partial^2 \theta(x,y)}{\partial x^2} + \frac{\partial^2 \theta(x,y)}{\partial y^2} = 0$$

$$\theta(x,y) = X(x)\,Y(y)$$

$$\frac{\partial^2}{\partial x^2} X(x)\,Y(y) + \frac{\partial^2}{\partial y^2} X(x)\,Y(y) = 0$$

$$-\frac{\partial^2}{\partial x^2} X(x) Y(y) = \frac{\partial^2}{\partial y^2} X(x) Y(y)$$

$$-Y(y) \frac{\partial^2}{\partial x^2} X(x) = X(x) \frac{\partial^2}{\partial y^2} Y(y)$$

$$\frac{-1}{X(x)} \frac{\partial^2 X(x)}{\partial x^2} = \frac{1}{Y(y)} \frac{\partial^2 Y(y)}{\partial y^2}$$

$$\frac{d^2 X}{\partial x} + \lambda^2 X = 0 \qquad \frac{\partial^2 Y}{\partial y} - \lambda^2 Y = 0$$

$$X(x) = C_1 \cos \lambda x + C_2 \sin \lambda x$$

$$Y(y) = C_3 e^{-\lambda y} + C_4 e^{\lambda y}$$

$$\theta(x,y) = (C_1 \cos \lambda x + C_2 \sin \lambda x)(C_3 e^{-\lambda y} + C_4 e^{\lambda y})$$

$$\theta(0,y) = 0 = C_1 (C_3 e^{-\lambda y} + C_4 e^{\lambda y})$$

$$C_1 = 0$$

$$\theta(x,0) = 0 = C_2 \sin \lambda x (C_3 + C_4)$$

if $C_2 = 0$ $\quad \theta = 0 \quad \forall \quad x, y$

$$C_3 = -C_4$$

$$\theta(L,y) = 0 = C_2 \sin \lambda L \, C_4 (e^{\lambda y} - e^{-\lambda y})$$

$$C_4 \neq 0$$

$$C_2 \neq 0$$

$\lambda = \frac{n\pi}{L}$ for integer $n > 0$

$$\theta = C_2 C_4 \sin \frac{n\pi x}{L} (e^{n\pi y/L} - e^{-n\pi y/L})$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\theta(x,y) = C_n \sin\frac{n\pi x}{L} \sinh\frac{n\pi y}{L}$$

infinite solutions

$$\theta(x,y) = \sum_{n=1}^{\infty} C_n \sin\frac{n\pi x}{L} \sinh\frac{n\pi y}{L}$$

$$\theta(x,W) = 1 \qquad C_n = \frac{2\left((-1)^{n+1}+1\right)}{n\pi \sinh(n\pi W/L)}$$

$$\boxed{\theta(x,y) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}+1}{n} \sin\frac{n\pi x}{L} \, \frac{\sinh(n\pi y/L)}{\sinh(n\pi W/L)}}$$